METROLOGICAL PROPERTIES OF A/D CONVERTERS UTILIZING HIGHER ORDER SIGMA–DELTA MODULATORS COMPARED WITH A/D CONVERTERS WITH MODULATORS OF FIRST ORDER

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Abstract

Time domain analysis is used to determine whether A/D converters that employ higher order sigma-delta modulators, widely used in digital acoustic systems, have superior performance over classical synchronous A/D converters with modulators of first order when taking into account their important metrological property which is the magnitude of the quantization error. It is shown that the quantization errors of delta-sigma A/D converters with higher order modulators are exactly on the same level as for converters with a first order modulator.

Keywords*: A/D converters, sigma-delta modulators, quantization errors.*

1. Introduction

A/D converters based on sigma – delta modulators are today widely employed in acoustic digital systems to convert analog sound signals into digital form. The digital samples are then recorded e.g. on CD, or transmitted through some kind of network. As digital signals, received or read from a CD, are converted back into an analog voltage to drive amplifiers and speakers or headphones, an analysis of such system very often takes into account jointly A/D and D/A conversions which could lead to misunderstandings if this is not clearly specified.

As in acoustic systems noise on the receiving side is of primary importance, frequency domain analysis is predominantly used for the analysis of such systems.

In several references [2, 3, 4, 5, 6, 7, 9] it is shown that noise shaping technique, implementation of comb-type decimalization output filters and other means which can, by using sigma – delta modulators of higher order, significantly improve the signal to noise ratio, resulting in digital acoustic systems of better quality.

There is no doubt that converters with first order delta modulators exhibit a periodical pattern of quantization error in the output digital words which causes the appearance of idle tones when the converter is used in audio systems, and that the higher order modulators eliminate the problem by providing a kind of input signal dithering.

However if the A/D converters based on sigma–delta modulators are used separately e.g. in measurement systems, it is not the level of noise, but rather the level of the limiting error which is of prime importance.

All the published analysis does not consider the crux of the matter which is the uncertainty of converter digital output due to the quantization error which is essential in metrological

converter applications, but concentrate on pointing out that modulators of higher order are much better as they provide a higher signal to noise ratio. This may lead to the conclusion that higher order delta modulators used in analog to digital converters somehow reduce the quantization error of the conversion results.

In order to determine the limiting errors of A/D converters that are used in measurement systems, time domain analysis can be used This analysis, apart from being a natural tool for such an investigation, also gives physical insight into the complex processes taking place in sigma-delta modulators, particularly those of higher orders.

The aim of this paper is to prove that by taking into consideration the quantization error there is no difference in performance of converters with higher order sigma-delta modulators when compared with modulators of first order.

Theoretical considerations have been supported by results obtained through the simulation of first and second order modulators action employing the MICROCAP circuits simulation program. Output pulses of both modulators were counted in the same period of time, which is equivalent to a decimation process.

Simulation results fully confirmed that when used in measurement systems, an A/D converter employing sigma – delta modulator of any order gives the same level of quantization error.

2. Theory

2.1. Frequency domain analysis

The block diagram of the simplified structure of an A/D converter with a first order sigma-delta modulator, as given in numerous papers [2, 3, 4, 5], is shown in Fig.1.
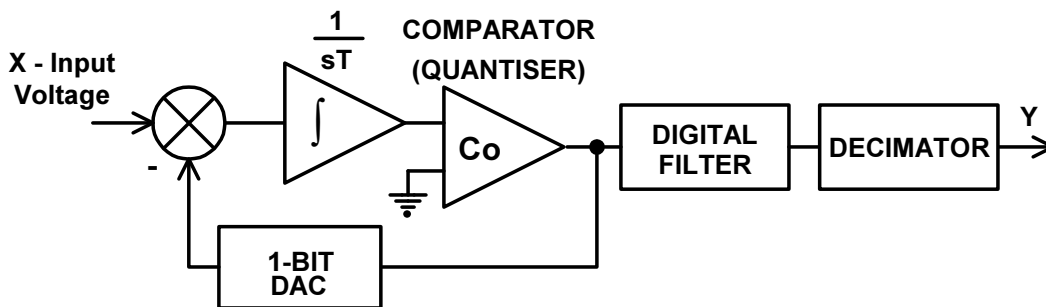

Fig. 1. Simplified structure of an A/D converter with first order modulator.

To explain the phenomenon of noise shaping, frequency domain analysis is used, based on the linearized, simplified structure of the first order sigma-delta modulator shown in Fig. 2.
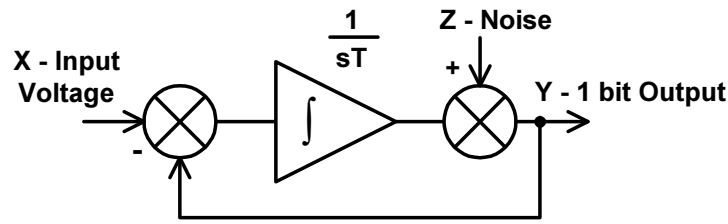
Fig. 2. Simplified basic structure of sigma-delta modulator of first order.

The output Y of the modulator can be considered as a train of one-bit samples of the analogue input X, and Z is the quantization noise produced by the quantizer (comparator).

The quantization noise can be considered as a random signal with frequency spectrum spread uniformly along the frequency axis.

The closed loop transfer function of the circuit can be evaluated from the equation,

$$\frac{X - Y}{sT} + Z = Y .$$ (1)

Assuming $Z = 0$, the transfer function of the input is equal to

$$G_S = \frac{Y}{X} = \frac{1}{1 + sT} ,$$ (2)

and if X is set to zero, the noise signal transfer function can be written as

$$G_Z = \frac{Y}{Z} = \frac{sT}{1 + sT} .$$ (3)

The integrator in the circuit loop acts as a low-pass filter for input X and high-pass filter for quantization noise Z.

The A/D converters with sigma-delta modulators sample the input signal with a frequency which greatly exceeds the value required to fulfil the Nyquist sampling theorem. Therefore the action of the high-pass filter for quantization noise pushes a large part of its spectrum beyond the signal frequency band.

Such an action, often referred to as noise-shaping, can be even more effective if sigma - delta modulators of higher order are employed. The simplified basic structure of a sigma-delta modulator of second order is presented in Fig. 3.
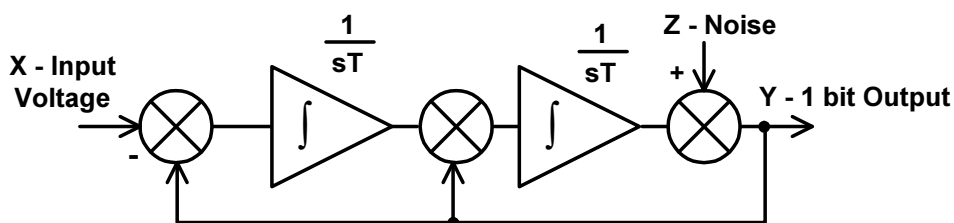

Fig. 3. Simplified basic structure of a sigma-delta modulator of second order.

The noise signal transfer function of the circuit, evaluated in a similar way as for the modulator of first order, can be expressed as

$$G_Z = \frac{Y}{Z} = \frac{s^2 T^2}{1 + s^2 T^2} \quad . \tag{4}$$

The digital filter shown in Fig.1, of low-pass transfer function, eliminates an even larger portion of the noise spectrum, so that A/D converters with second and higher order modulators are considered better than converters with first order modulators; they also provide stronger noise-shaping, which in effect produces a higher signal to noise ratio. The digital filter is supposed to also act as a decimator, reducing the number of samples, and hence also reducing the quantization noise level.

To illustrate the noise-shaping phenomenon very often diagrams such as the one given as in Fig. 4 are used.

What is inconsistent in the above reasoning is the assumption that the digital low-pass filter acts exactly on the output of the sigma-delta modulator, reducing significantly quantization noise, due to the noise shaping phenomenon.

The digital filter, placed in such position, cannot perform any other operation except change the distribution of output pulses of the modulator. The density of pulses must remain unchanged, as it contains information on the input signal magnitude. Therefore decimation must take place before any digital filtering is applied.

Decimation in this case should be understood as the simple counting of the number of pulses during a chosen time interval, which determines the decimation ratio and resolution of the output digital multi-bit samples of the input.
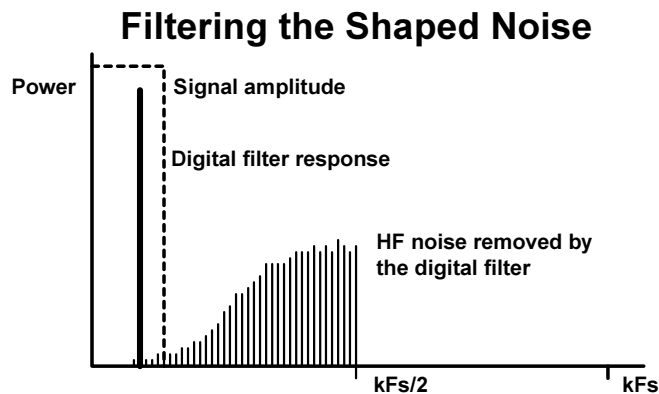

Fig. 4. Diagram illustrating how the digital filter eliminates the shaped noise [4].

The counter itself acts as a low-pass filter reducing the quantization noise produced by the coarse process of quantization performed by the comparator. It is therefore doubtful whether any consecutive digital filtering would improve the performance of the A/D converter, taking into account the output samples quantization error.

The noise shaping action performed by consecutive digital filtering can be then associated with the action of digital to analogue conversion [9, 10, 11] taking place at the output end of the digital audio system.

It is widely accepted and proven [6] that A/D converters with first order sigma-delta modulators sometimes exhibit a periodic pattern of quantization error on 1-bit level in the

output samples, which causes the appearance of idle tones when the converter is used in audio systems. The magnitude of the idle tone is small, but can cause annoying distortion when the total magnitude of the acoustic signal is small.

The use of A/D converters with modulators of higher order reduces the problem of idle tones, which is perhaps the reason why they are widely advertised as superior to the simpler converters with first order modulators.

2.2. Time domain analysis

The time domain analysis can be used by tracing the origin of sigma-delta converters to an asynchronous type of voltage to frequency converter which utilizes the concept of charge balance [1, 3]. The simplified circuit diagram of such an asynchronous voltage to frequency converter is shown in Fig. 5.

The integrator capacitor is charged by the current ($I + i$) during time τ and discharged by the current ($I - i$) during the remainder of the time period T.
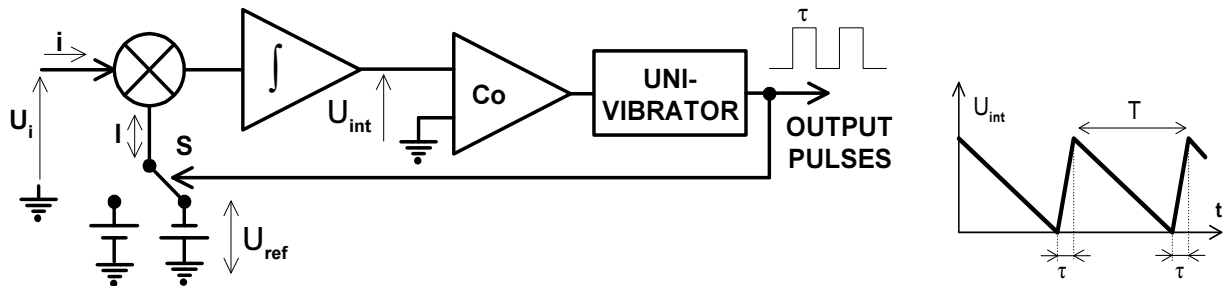

Fig. 5. Simplified circuit diagram of an asynchronous voltage to frequency converter
Co - Comparator,   **S -** Switch.

By comparing the charges entering and leaving the integrator capacitor, at the instant of time when the integrator output voltage (U_{int}) reaches zero, it can be shown that the frequency of pulses appearing at the converter output is given by

$$f = \frac{1}{T} = \frac{1}{2\tau} + \frac{i}{2I\tau},$$

(5)

where τ is the duration of pulse generated by the univibrator and $I = U_i /R$, where R denotes the input resistance of the integrator.

The asynchronous voltage to frequency converter can be used to build an A/D converter. The output pulses are counted during a fixed time interval, which is also the sampling period of the converter input signal.

The practical difficulty with such a converter structure is that a highly stable clock must be used to set the counting time interval, as the number recorded by the counter depends not only on the incoming frequency, but also directly on the counting time interval.

The necessity of using a highly stable clock can be avoided if the structure of the asynchronous voltage to frequency converter is modified by adding a clock to the circuit which, together with the comparator, controls the switch **S**.

With such a modification a synchronous voltage to frequency converter is obtained as shown in Fig. 6 and may be designated as a *sigma-delta modulator of first order.*
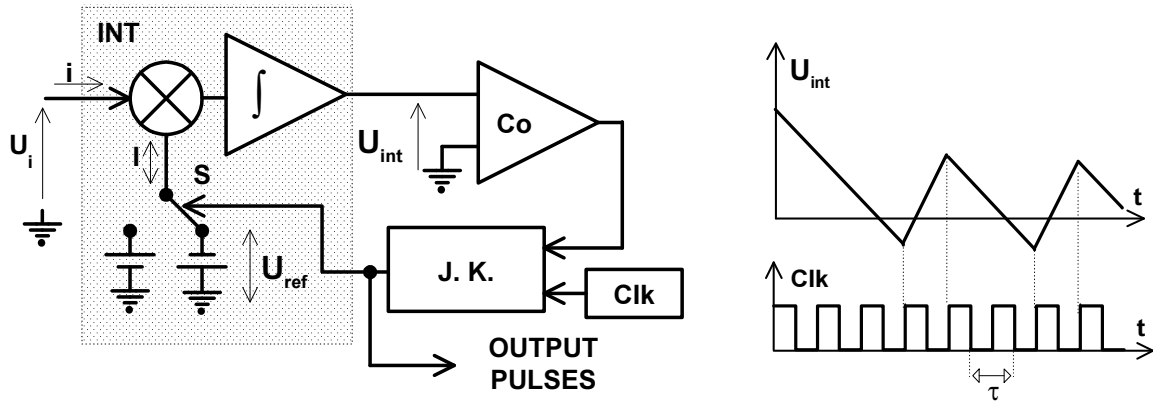

Fig. 6. Simplified circuit diagram of a synchronous voltage to frequency converter (delta modulator of first order) **Co -** Comparator, **J.K. -** JK-MS–Flip flop, **Clk -** Clock, **S -** Switch.

Because now the counting time interval can be set as a fixed number **M** of the same clock pulses, stability of the clock is no longer required to obtain an accurate A/D converter.

The main difference in operation, compared with an asynchronous voltage to frequency converter, is that the output voltage of the integrator can now go more or less below the zero reference level of the comparator since the output pulse of the comparator is not synchronized with the rising edges of the clock pulses.

This is why, to analyze the synchronous voltage to frequency converter shown in Fig. 6, the charge balance condition, formulated for the moments at which the value of integrator output voltage equals zero, cannot be fulfilled exactly.

However, taking into consideration a certain number of clock pulses **M**, equivalent to a time interval $T = M\tau$, and assuming that N output pulses from the converter appeared during this period, the following charge balance equation can be formulated

$$(I - i)N\tau = (I + i)(M - N)\tau \pm \Delta Q, \tag{6}$$

where $\Delta Q \leq 2I\tau$ is the charge transferred during a single pulse.

From (6) the number N of pulses can be evaluated as

$$N = \frac{M}{2} + \frac{i}{2I}M \pm 1, \tag{7}$$

where $i = \dfrac{U_i}{R}$ and R denotes the input resistance of the integrator.

It is interesting to note that defining an equivalent frequency f_e of output pulses as the ratio of N to the period of counting $T = M\tau$, and neglecting ± 1, which is justified for large values of M, an expression identical to that for the asynchronous voltage to frequency converter can be obtained

$$f_e = \frac{N}{M\tau} = \frac{1}{2\tau} + \frac{U_i}{2IR\tau}. \tag{8}$$

Modulators of higher order have more than one integrator unit (**INT** in Fig. 6), connected in series, as shown in Fig. 7 for a third order modulator.
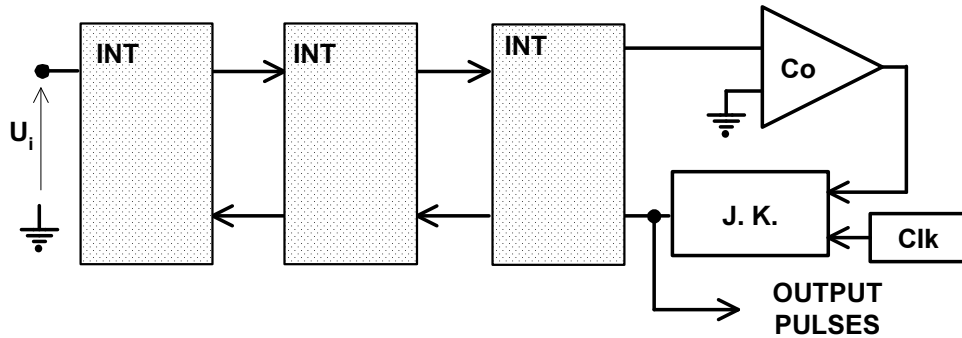

Fig. 7. Simplified circuit diagram of a third order sigma-delta modulator.

Since the charge balance condition (6) has been evaluated for the output of the last integrator which triggers the comparator, it follows that an identical relation should hold also for modulators of any order higher than first, which are built with several integrator units prior to the final one.

The multi-bit output of an A/D converter, utilizing sigma delta modulators, is obtained through a process of decimation, which in principle is the counting of the output pulses of the sigma-delta converter during a fixed period of time determined by the chosen number **M** of clock pulses. The principle of such an A/D converter is shown in Fig. 8.
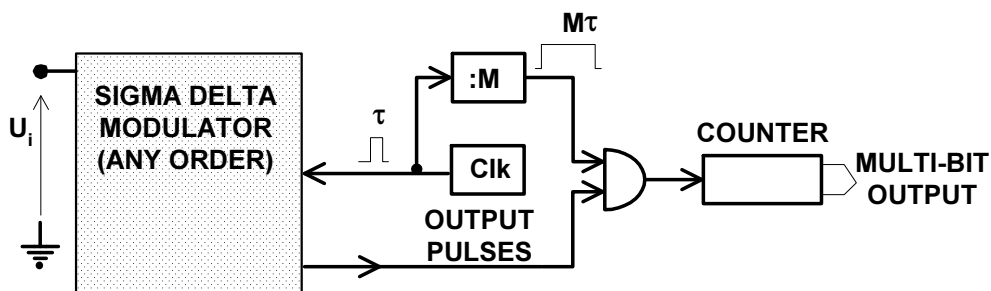

Fig. 8. Principle of an A/D converter with sigma-delta modulator **Clk** - Clock,   **:M** - Frequency divider.

The ±1 count in equation (7) represents one pulse, or one-bit uncertainty of the conversion result, and does not depend on the number of integrators within the sigma-delta converter structure i.e. on the sigma-delta modulator order.

It can thus be concluded that the quantization errors of delta-sigma analog to digital converters with higher order modulators are exactly on the same level as for converters with a first order modulator, and depend only on the oversampling ratio of the A/D converter, determined by the number of clock pulses *M*.

For applications in instrumentation systems no further improvements can be obtained, but in digital audio systems where not the quantization error but the signal to noise ratio is of prime importance, further reduction of noise level can be obtained by including sophisticated digital filtering in digital-to-analogue converters [9-11].

3. Experiments

To verify the above conclusion a simulation experiment was carried out, in which the performance of first and second order modulators were compared using TRANSIENT analysis of the MICROCAP [8] circuit simulator.

The output graphs, shown in Fig. 9 and Fig. 10 were obtained for modulators of first and second order.

The numerical results of simulation have been recorded in separate files and used to simulate the decimation process. The number N of output pulses (equation 7) for both modulators was counted during the period of $M = 1000$ consecutive clock pulses, for different values of the input voltage. The results of the counting, which are equivalent to the multi-bit output samples of the A/D converters built with sigma-delta modulators of first and second order respectively, are given in Table 1.

Table 1. Number of output pulses of first and second order modulators during $M = 1000$
clock pulses fordifferent input voltage values.

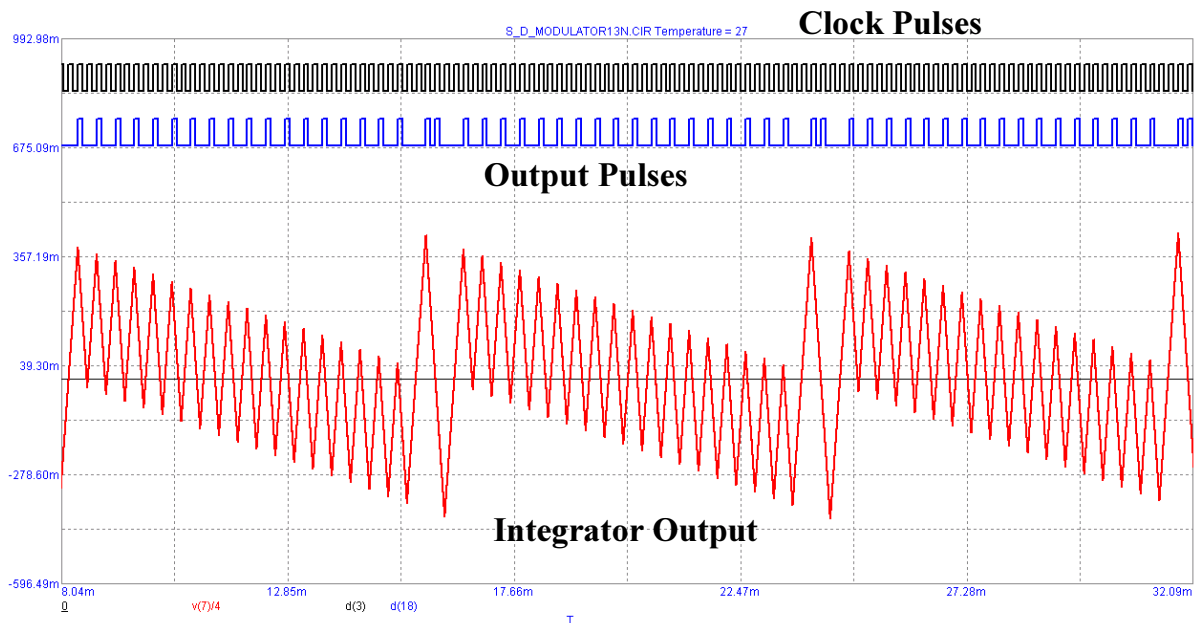| Input voltage [V]	-3	-2	-1	0	1	2	3
N – Number of output pulses First order modulator	124	249	374	499	624	749	874
N – Number of output pulses Second order modulator	125	250	375	499	624	748	873

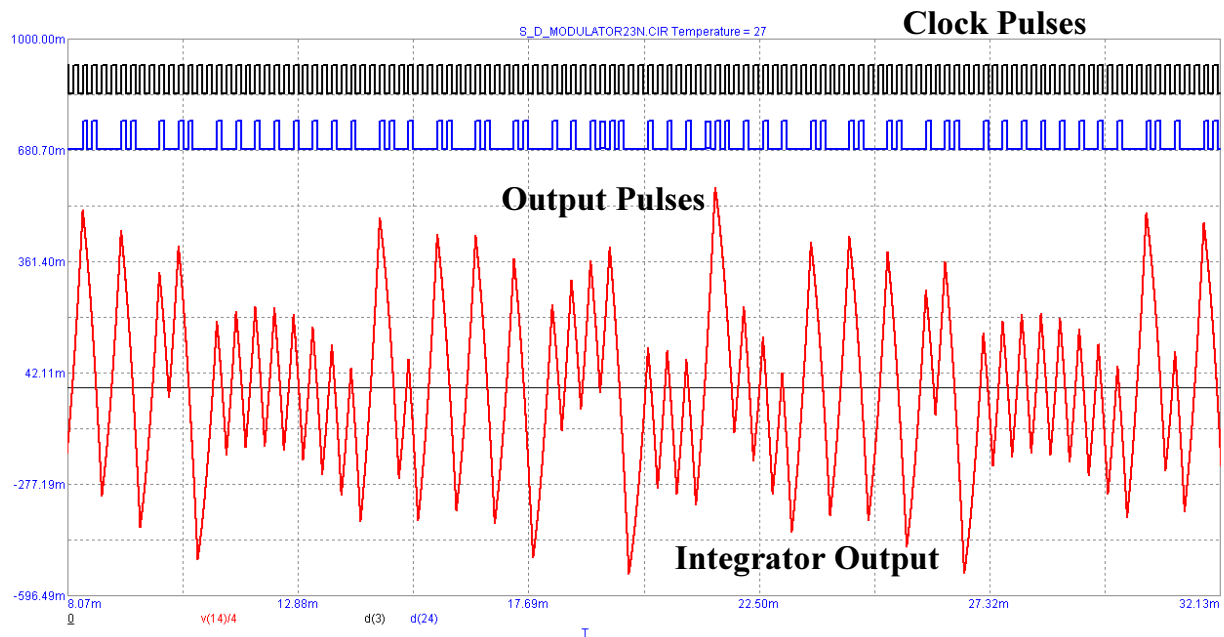Fig. 9. Output waves of first order sigma-delta modulator.

Fig. 10. Output waves of second order sigma-delta modulator.

4. Conclusions

On the basis of results given in Table 1, it can be seen that in no case is the difference in pulse count between a first order and a second order modulator greater than one pulse. This means that in both cases the quantization error value of an A/D converter built with a sigma-delta modulator either of first or second order is on the same level of one pulse count thus confirming that equation (7) is valid for modulators of any order; it also proves that using a modulator of any order has no influence on the sigma-delta analog to digital converter quantization error value.

Comparing the shapes of the integrator output waves, given in Figs. 9 and 10, it can be noticed that converters with first order delta modulators exhibit a distinct periodical pattern, whereas this is not apparent in the case of a modulator of second order. It means that the quantization error of the A/D converter with first order modulator also exhibits a periodical pattern and such an error in the digital output would cause the appearance of idle tones, on one bit level, when the converter is being used in audio systems. Output pulses of a second order modulator appear in a much less regular pattern; therefore use of A/D converters with modulators of higher order in such systems can be justified, as they reduce the problem of idle tones by providing a kind of input signal perturbation. However, in instrumentation systems for example, the order of the sigma-delta modulator in the structure of an A/D converter has no practical influence on the system performance.

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