Abstract

This paper deals with the experimental validation of the suitability of the method for measuring radial variations of components on the process tool. The tests were conducted using a computerized PSA6, which was compared to a Talyrond 73. The results of measurement of roundness deviations as well as roundness profiles were analyzed for a sample of 70 shafts. The roundness deviations were assessed by determining the experimental errors, while the profiles obtained with the tested device were compared to those registered by the reference device using three correlation coefficients.

Keywords: roundness, experimental method error, process tool.

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1. Introduction

The method for measuring roundness profiles on the process tool which bases on the variations in the component radius belongs to a group of non-reference methods. It requires fixing a component in the centres, rotating it and registering the variations in the radius in the rotation angle function by means of a probe fitted perpendicular to the axis of rotation [1, 2, 3].

The measurement data may contain an error, which is related to the type of probe applied and the accuracy with which the center holes were made. The method error was determined theoretically [4]. It can also be estimated experimentally by comparing the results obtained with the tested instrument to those from the reference instrument. This paper discusses the results of statistical tests and calculations for the analyzed and reference instruments, PSA6 and Talyrond 73, respectively. The assessment was made using the experimental error of roundness deviation and the correlation-based comparison of the measured roundness profiles. In statistical testing, we used a sample of 70 ground shafts with center holes selected at random from a batch. The measurements were conducted under laboratory conditions at the Kielce University of Technology, applying equipment suitable for measurement of form profiles.

The sequence and range of the calculations were as follows:

a. Determination of the experimental error of the method for measuring roundness profiles, according to the principles of statistical inference, taking roundness deviations into consideration:
   − the procedure for the estimation and test of significance for the mean value of the experimental error [1, 5],
   − the procedure for the estimation and test of significance for the variances and mean deviations of the experimental error [1, 5],
the estimation of the confidence interval of a single method error and measurement accuracy,
the procedure for the test of concordance between the distribution of the method error in a population with the theoretical distribution [1, 6]
b. Statistical comparison of roundness profiles using the correlation calculus:
comparison of roundness profiles by means of cross-correlation coefficients,
comparison of roundness profiles by means of Pearson’s linear correlation coefficients,
comparison of roundness profiles by means of Spearman’s rank correlation coefficients.

First, the reference instrument, Talyrond 73, was used to measure the roundness profile of each component of a sample. In this way, a real roundness profile was obtained. The measurement results for a given roundness profile in the digital form and the values of the roundness deviation, ΔZ, and the amplitudes of the particular harmonics of this profile were transmitted to the memory of the test-stand computer.

Statistical inference, i.e. drawing conclusions on the properties of the general population basing on the results obtained for a sample or samples drawn from this population, requires estimating the values of the parameters of the distribution (point estimation), determining the confidence intervals (parameter estimation) or testing statistical hypotheses. To reduce the probability of errors to a minimum, it was necessary to:

select an appropriate statistical method according to the data concerning the analyzed properties and the tests to be conducted,
use a representative sample,
strictly follow the procedure of each statistical method,
apply the statistical methods selected for each test only once,
maintain the assumed level of significance throughout the entire statistical hypothesis test,
maintain the assumed level of confidence when determining the confidence interval.

2. Experimental error

The basic method for identifying the accuracy of a measurement instrument is to compare its results with those obtained by means of a reference instrument [1]. One of the most important parameters used for this purpose is the experimental error (ΔEBM) described by the following relationship:

\[ ΔEBM = \frac{x_i - y_i}{y_i}, \]  

(1)

where: $ΔEBM$ – experimental error,
$x_i$ – measurement result obtained with the tested instrument;
$y_i$ – measurement result obtained with the reference instrument.

The calculated experimental errors were used to determine the accuracy of the analyzed instrument and, accordingly, validate its suitability for certain applications.

Table 1 shows ranges of relative method errors established in our previous research and the corresponding applications.
Table 1. Ranges of relative method errors in surface texture measurement and the corresponding applications [1].

<table>
<thead>
<tr>
<th>Measurement accuracy range [%]</th>
<th>Type of application</th>
</tr>
</thead>
<tbody>
<tr>
<td>2% ÷ 5%</td>
<td>Measurement of standards: roughness, waviness, form profiles</td>
</tr>
<tr>
<td>5% ÷ 15%</td>
<td>Scientific research</td>
</tr>
<tr>
<td>10% ÷ 25%</td>
<td>Measurement of surface texture under industrial conditions</td>
</tr>
</tbody>
</table>

3. Statistical determination of the method error using roundness deviations

A sample consisting of 70 ground shafts was used to establish the experimental error through statistical testing. The analysis included:
- the estimation and test of significance for the mean value,
- the estimation and test of significance for the variances and mean deviations,
- the estimation of the confidence interval of a single method error for the assumed confidence,
- the determination of the concordance of the distribution of the method error in a population with the theoretical distribution.

3.1. Procedures for the estimation and test of significance for the mean value of the experimental error

In order to compare the mean values of the experimental error, it was necessary to measure roundness deviations in the same cross-sections using three different instruments. First, the value of the roundness deviation was established by means of the reference instrument, i.e. the Talyrond 73, with higher spindle rotation accuracy (spindle runout of 20 nm). Then, the experimental errors were calculated. Their mean values were determined using the following relationship:

\[
\overline{\Delta EBM} = \frac{1}{n} \sum_{i=1}^{n} \Delta EBM_i,
\]

where: 
- \( n \) – number of samples,
- \( \Delta EBM_i \) – relative method error of the roundness deviation for each element of the sample.

The mean value of the method error in the analyzed population was estimated using the following procedure:

a. Determine the method error including the roundness deviation of the profile measured at a computerized test stand,

b. Estimate the mean value of the method error,

c. Estimate the interval of confidence for the mean value of the method error with normal distribution and the unknown mean deviation using the following formula:

\[
\left( \overline{\Delta EBM} - u_p \cdot \frac{s}{\sqrt{n}} ; \overline{\Delta EBM} + u_p \cdot \frac{s}{\sqrt{n}} \right),
\]

where: 
- \( \overline{\Delta EBM} \) – the mean value of the method error,
- \( s \) – the calculated mean deviation,
- \( u_p \) – the quantile of the normal distribution read from the tables in Refs.[1, 6, 7, 8].
To explain whether or not the divergence between the mean values is random, it was necessary to conduct a test of the means for a case when the values of the mean square errors are unknown. The mean values could be thus compared by calculating the value of $t_s$ from:

$$
 t_s = \frac{x_1 - x_2}{s_z \cdot \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}},
$$

where:
- $x_1, x_2$ - the mean values being compared,
- $n_1, n_2$ – number of measurements,

$$
 s_z = \sqrt{\frac{(n_1 - 1) \cdot s_1^2 + (n_2 - 1) \cdot s_2^2}{(n_1 - 1) + (n_2 - 1)}},
$$

where: $s_1, s_2$ – mean square errors of the measured values of $x_1, x_2$

The calculated value of $t_s$ was compared with its critical value, $t(P, k)$ read from the tables in Ref. [1]. The critical value is determined when $P = 0.95$ and the number of degrees of freedom is calculated from the following relationship.

$$
 k = n_1 + n_2 - 2.
$$

If $t_s$ exceeds the critical value, it can be assumed that the divergence is significant; if not, the divergence is random (insignificant).

### 3.2. Procedures for the estimation and test of significance for the variances and mean deviations of the experimental errors

When establishing the relationships between the variances, one needs to estimate and test the significance for the variances. This is particularly important when the accuracy of instruments is checked.

The procedure to estimate the population variance for a case when the mean value of the method error is unknown involves:

a. calculating the variances value according to the formula:

$$
 s^2 = \frac{1}{n - 1} \cdot \sum_{i=1}^{n} (\Delta EBM_i - \bar{\Delta EBM})^2,
$$

where: $n$ – number of samples,
- $\Delta EBM_i$ – the method error for the measurement of an element in the sample,
- $\bar{\Delta EBM}$ – the estimated mean method error for the sample.

b. calculating the value of the coefficient $F$ using the following relationship:

$$
 \frac{s_1^2}{s_2^2} = F > 1.
$$

The calculated value of the coefficient $F$ was compared with the critical value read from the tables in Ref. [1]. If the value of $F$ is greater than the critical value, the divergence between the analyzed variances is significant. However, when $F$ is less than the critical value, the divergence is assumed to be random (insignificant).
3.3. Estimation of the confidence interval of a single method error and measurement accuracy

The confidence interval of a single method error for the assumed level of significance was read from the normal distribution tables. The confidence interval was calculated using the following relationships:

\[ (\Delta EBM - u_p \cdot s, \quad \Delta EBM + u_p \cdot s), \]

where:
- \( \Delta EBM \) – the mean value of the method error,
- \( u_p \) – the quantile of the normal distribution,
- \( s \) – the mean square deviation of the method error.

The measurement accuracy of the analyzed methods was determined using the following relationship:

\[ DP = \left| \Delta EBM \pm u_p \cdot s \right|_{\text{max}}. \]

Formula (10) was used to qualitatively assess the measurement accuracy of each method. It included roundness deviations of the components from a given statistical sample, which were measured twice: first with the reference instrument, and then with the tested instrument. Accuracy defined in this way includes experimental errors related to the systematic and random errors of the measurement of this deviation.

3.4. Procedure for the test of the concordance between the distribution of the method error in a population with the theoretical distribution

All the considerations were based on the assumption that the results of the experiment are distributed normally. The doubts concerning the normality of distribution will be finally dissolved if the procedure for the tests of the concordance between the method error distribution and the theoretical distribution is applied. In this analysis, we used one of the most popular tests determining the level of concordance with the normal distribution – test \( \chi^2 \) [1, 3].

3.5. Assessing the results of statistical tests for the method error

Table 2 includes results of the statistical tests of the method error established for the measurement of radius variations on a process tool for a sample of 70, estimation of the tests, estimation of the confidence intervals and tests of concordance between the method error distribution and the theoretical distribution.

The mean values of the relative experimental error of the method and the confidence intervals of a single value of the error indicate that the accuracy of measurement of roundness deviations for the analyzed samples ranges from 10% to 25% [1].

Table 2 shows results of statistical testing of the experimental method error calculated with respect to the Talyrond 73. The significance test for the mean values and the variances indicates that the divergence between the values calculated on the basis of the randomized test and the critical values is random. The test of concordance with the theoretical distribution confirms that the results of the experimental error are in agreement.

The statistical testing of the experimental error made it possible to calculate the accuracy of the method for measuring roundness profiles on the process tool based on the variations in component radius. The accuracy was approximately 18%, which confirms the instrument suitability for analyzing surface structure under industrial conditions.
Table 2. Results of the statistical determination of the experimental error for the PSA 6 based on the results obtained by means of the Talyrond 73.

<table>
<thead>
<tr>
<th>Sample type</th>
<th>Ground shafts</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of samples</td>
<td>70</td>
</tr>
<tr>
<td>Observed value of the method error</td>
<td>( \Delta EBM_{min} ) 0.000</td>
</tr>
<tr>
<td></td>
<td>( \Delta EBM_{max} ) 0.253</td>
</tr>
<tr>
<td>Mean value</td>
<td>0.081</td>
</tr>
<tr>
<td>Confidence interval for ( P=0.95 ) (( \mu_p=1.96 ))</td>
<td>0.081±0.013</td>
</tr>
<tr>
<td>Test of significance for the mean value</td>
<td>random divergence</td>
</tr>
<tr>
<td>Test of significance for the variances</td>
<td>random divergence</td>
</tr>
<tr>
<td>Mean deviation ( s )</td>
<td>0.056</td>
</tr>
<tr>
<td>Test of concordance with the theoretical distribution</td>
<td>The error distribution in concordance with the theoretical distribution</td>
</tr>
<tr>
<td>Method accuracy MA [%]</td>
<td>18 %</td>
</tr>
</tbody>
</table>

4. Method for comparing roundness profiles by means of the correlation calculus

4.1. Comparison of roundness profiles using cross-correlation coefficients

The analysis and estimation of the experimental method error were used to assess the suitability of an instrument for accurate measurement. It is extremely important to determine how similar the measured profiles are, because, theoretically, an instrument may provide us with approximate results despite the fact that a completely different roundness profile is registered. It is possible to visually compare two results obtained for the same cross section with different measurement instruments. The comparison, however, is only of qualitative character. To compare the measured profiles quantitatively, one needs to use the correlation calculus [1], by determining the cross-correlation function.

It can be represented using the relationship for the so-called coefficient of concordance:

\[
r(\phi_0) = \frac{2 \int_{0}^{2\pi} Z_T(\phi)Z_W(\phi + \phi_0)d\phi}{\int_{0}^{2\pi} Z_T^2(\phi)d\phi + \int_{0}^{2\pi} Z_W^2(\phi)d\phi},
\]

where: \( Z_T(\phi) \) – profile measured by applying the tested method, \( Z_W(\phi) \) – profile measured using the reference method, \( \phi_0 \) – shift between the measured profiles.

The coefficients can be used to compare even a single measurement result. They may range from –1 to 1. When the cross-correlation coefficient is negative, there is definitely no agreement between the measured profiles. Positive coefficients can be estimated basing on the rules provided by I. P. Guilford [1].

For the entire sample (\( n = 70 \)), we determined:
– the mean value and confidence interval,
– the confidence interval for a single cross-correlation coefficient.

Using the calculated mean values of the coefficients of concordance between the compared roundness profiles and the estimated confidence intervals for a single value of the coefficient
one can assume that the correlation between the compared roundness profiles, i.e. ones measured with the radial method and ones obtained with the reference instrument is very high. This is confirmed also through qualitative (visual) comparison of these profiles (Figs. 1 and 2).

Table 3. Results of statistical tests of cross-correlation coefficients for the pair of instruments: the PSA 6 and the Talyrond 73

<table>
<thead>
<tr>
<th>Sample type</th>
<th>Ground shafts</th>
</tr>
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<tbody>
<tr>
<td>Number of samples</td>
<td>70</td>
</tr>
<tr>
<td>Observed value</td>
<td></td>
</tr>
<tr>
<td>min</td>
<td>0.9313</td>
</tr>
<tr>
<td>max</td>
<td>0.9998</td>
</tr>
<tr>
<td>Mean value</td>
<td>0.995</td>
</tr>
<tr>
<td>Confidence interval for P=0.95 ($u_p=1.96$)</td>
<td>0.995±0.002</td>
</tr>
<tr>
<td>Mean deviation $s$</td>
<td>0.009</td>
</tr>
<tr>
<td>Confidence interval for the variances</td>
<td>0.009±0.00002</td>
</tr>
</tbody>
</table>

Figures 1 and 2 show the visual comparison of roundness profiles measured using the reference instrument (Talyrond 73) and the analyzed instrument (PSA6). The results illustrated below were obtained for three components with different values of the coefficients of concordance.

Fig. 1 Visual comparison of two roundness profiles in rectangular coordinates measured with two different instruments (the Talyrond 73 – solid line, the PSA 6 – dotted line: a) comparison for component No 16 (cross-correlation coefficient 0.9994), b) comparison for component No 26 (cross-correlation coefficient 0.9989), c) comparison for component No 68 (cross-correlation coefficient 0.9951)
4.2. Comparison of roundness profiles using Pearson’s linear correlation coefficients

An alternative solution for a cross-correlation function is to apply the Pearson’s linear correlation function to assess the concordance of profiles. The comparison required changing the coordinates of the measurement points into amplitudes and phase shifts for harmonics from 2 to 15. That was possible by applying a Fast Fourier Transform. The estimation was carried out using the obtained values of amplitudes of the particular harmonics. Establishing the concordance between profiles by means of phase shifts was less important as this testifies to the repeatability of positioning of a component in an instrument [1].

Once the values of the amplitudes of the harmonics from the two instruments (the reference measurement instrument and the PSA 6) were grouped into 14 sets for each
harmonic number, they were statistically analyzed, which involved establishing the coefficient of correlation between the amplitudes of the harmonics obtained by using the reference method and the tested method. Then, the matrix of the correlation coefficients was calculated.

Pearson’s correlation coefficients were calculated using the following relationship:

\[ r = \frac{S_{XYn}}{s_{Xn} \cdot s_{Yn}}, \]  

(12)

where:  

- \( s_{XYn} \) - covariance of the \( X_i, Y_i \) sets,  
- \( s_{Xn} \) - mean deviation for the values of the \( X_i \) set,  
- \( s_{Yn} \) - mean deviation for the values of the \( Y_i \) set.

\[ S_{XYn} = \frac{1}{n} \sum_{i=1}^{n} (C_{Xn_i} - \overline{C}_{Xn})(C_{Yn_i} - \overline{C}_{Yn}), \]  

(13)

where:  

- \( C_{Xn_i} \) - values of the amplitudes of the particular harmonics for profile \( Z_p (\phi) \)  
- \( \overline{C}_{Xn} \) - mean value of the amplitudes of the particular harmonics for profile \( Z_p (\phi) \)  
- \( C_{Yn_i} \) - values of the amplitudes of the particular harmonics for profile \( Z_a (\phi) \)  
- \( \overline{C}_{Yn} \) - mean value of the amplitudes of the particular harmonics for profile \( Z_a (\phi) \)

The SADCOM program was used to calculate the correlation matrices. The results are tabularized in Table 4. Using the I.P. Guilford table, one can determine the relationships between the obtained correlation coefficient and the degree of correlation between the obtained roundness profiles.

Table 4. Pearson’s correlation matrix for measurements by applying the tested method and the reference measurement instrument Talyrond 73

<table>
<thead>
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<td>0.919</td>
<td>0.94</td>
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</tr>
<tr>
<td>13</td>
<td>0.79</td>
<td>0.82</td>
<td>0.84</td>
<td>0.86</td>
<td>0.88</td>
<td>0.9</td>
<td>0.92</td>
<td>0.94</td>
<td>0.96</td>
<td>0.98</td>
<td>0.8</td>
<td>0.82</td>
<td>0.84</td>
<td>0.86</td>
<td>0.88</td>
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</tr>
<tr>
<td>14</td>
<td>0.32</td>
<td>0.35</td>
<td>0.38</td>
<td>0.41</td>
<td>0.44</td>
<td>0.47</td>
<td>0.5</td>
<td>0.53</td>
<td>0.56</td>
<td>0.59</td>
<td>0.62</td>
<td>0.65</td>
<td>0.68</td>
<td>0.71</td>
<td>0.74</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>0.10</td>
<td>0.216</td>
<td>0.66</td>
<td>0.688</td>
<td>0.713</td>
<td>0.736</td>
<td>0.759</td>
<td>0.782</td>
<td>0.805</td>
<td>0.828</td>
<td>0.851</td>
<td>0.874</td>
<td>0.897</td>
<td>0.919</td>
<td>0.94</td>
<td></td>
</tr>
</tbody>
</table>

In addition, the hypotheses were verified for each correlation coefficient by calculating the statistic according to:

\[ t = \frac{r_n}{\sqrt{1 - r_n^2}} \sqrt{n - 2}, \]  

(14)

where:  

- \( r_n \) – estimated correlation coefficient,  
- \( n \) – number of samples.

After assuming the probability (P = 0.95) and the number of degrees of freedom \( n - 2 \), we read the critical values, \( t_{kr} \), and compared them to the statistic, \( t \). It was then possible to
define whether or not there exists any correlation. In the correlation matrix, the highlighted numbers indicate that the correlation occurs, while those in a white background represent null correlation.

4.3. Comparison of roundness profiles using Spearman’s rank correlation coefficients

Another function used to compare measured roundness profiles is the Spearman correlation. Since the method has previously proved to be effective, it was used for this analysis. The basic definition of Spearman’s correlation function is as follows: if the compared properties can be ordered increasingly, then it is possible to apply a rank correlation function. This was particularly significant in the experimental tests conducted as part of this analysis. Determining Spearman’s rank correlation coefficients for measurement of different components will imply that there is a correlation between the obtained roundness profiles.

The procedure to derive Spearman’s coefficients of the rank correlation between the amplitudes of the particular compared roundness profiles was as follows:

- determining the amplitudes of each harmonic in the X and Y sets, from the highest to the lowest according to variable \( Z_T \), and then according to another variable, i.e. \( Z_W \),
- for each pair of ranks \( Z_T \) and \( Z_W \), determining the difference \( D \) by subtracting the lower rank from the higher one,
- assessing a rank correlation coefficient using the formula

\[
r_s = 1 - \frac{6D_n}{n(n^2 - 1)},
\]

where: \( D_n \) – sum of squares of differences between the ranks determined by means of relationship (16)

\[
n \quad \text{number of samples.}
\]

\[
D_n = \sum_{i=1}^{n} (r_{xi} - r_{yi})^2,
\]

where: \( r_{xi} \) – rank of an element of the \( X_i \) set,

\( r_{yi} \) – rank of elements of the \( Y_i \) set.

As shown in Table 5, the calculated coefficients of the rank correlation between the values of amplitudes of the particular harmonics for each sample were given in the form of the matrix of Spearman’s correlation coefficients. In the matrix, the null correlation results were highlighted.

Table 5. Spearman’s correlation matrix
4.4. Estimating the results of statistical testing for the compared roundness profiles using the correlation calculus

Pearson’s correlation coefficients were calculated for each set of amplitudes of the compared roundness profiles using a special computer program, SADCOM [9]. The profiles were measured by means of:
- non-reference instruments equipped with ROFORM and CYFORM software,
- reference instruments equipped with the SAJD software,
- ZEISS coordinate machines with the CALYPSO software.
- Some of the calculation results obtained by applying Pearson’s and Spearman’s correlation coefficients for the amplitudes of each harmonic are provided in Table 6.

Table 6. Pearson’s and Spearman’s coefficients of correlation between the two instruments, the PSA6 and the Talyrond 73, for the amplitudes of each harmonic

<table>
<thead>
<tr>
<th>Harmonic number</th>
<th>Pearson’s coefficient</th>
<th>Spearman’s coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.987</td>
<td>0.963</td>
</tr>
<tr>
<td>3</td>
<td>0.998</td>
<td>0.995</td>
</tr>
<tr>
<td>4</td>
<td>0.998</td>
<td>0.996</td>
</tr>
<tr>
<td>5</td>
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<td>0.994</td>
</tr>
<tr>
<td>6</td>
<td>0.996</td>
<td>0.995</td>
</tr>
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<td>7</td>
<td>0.987</td>
<td>0.986</td>
</tr>
<tr>
<td>8</td>
<td>0.992</td>
<td>0.992</td>
</tr>
<tr>
<td>9</td>
<td>0.978</td>
<td>0.967</td>
</tr>
<tr>
<td>10</td>
<td>0.975</td>
<td>0.968</td>
</tr>
<tr>
<td>11</td>
<td>0.963</td>
<td>0.955</td>
</tr>
<tr>
<td>12</td>
<td>0.974</td>
<td>0.958</td>
</tr>
<tr>
<td>13</td>
<td>0.917</td>
<td>0.912</td>
</tr>
<tr>
<td>14</td>
<td>0.955</td>
<td>0.937</td>
</tr>
<tr>
<td>15</td>
<td>0.913</td>
<td>0.909</td>
</tr>
</tbody>
</table>

From Table 6 it is clear that all the analyzed relationships are correlated, which was denoted by the plus sign (+) at each correlation coefficient. The values of the correlation coefficients show that there is a very high or high correlation between the amplitudes of the harmonics that are dominant for the compared roundness profiles and the amplitudes of the harmonics that have a significant effect on the profile form ($n = 2\text{÷}10$).

For the other amplitudes of each harmonic, the correlation is in most cases very high, high and moderate.

5. Conclusion

The statistical tests show that there exists a high correlation between the roundness deviations as well as the roundness profiles measured with two different methods. The 18 % accuracy confirms that the analyzed method is suitable for measuring surface texture under industrial conditions. Visual comparison of roundness profiles also shows that the tested instrument can be used for accurate measurements. The correlation between measured profiles was very high when the profiles were compared using a function of correlation, i.e. cross-correlation, Pearson’s correlation and Spearman’s correlation. The tests confirm that the analyzed method can be effectively used for measuring roundness profiles under industrial conditions.
References


