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QUANTITATIVE COMPARISON OF CYLINDRICITY PROFILES MEASURED WITH DIFFERENT METHODS USING LEGENDRE-FOURIER COEFFICIENTS

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Abstract

The paper discusses a method of quantitative comparison of cylindricity profiles measured with different strategies. The method is based on applying so-called Legendre-Fourier coefficients. The comparison is carried out by computing the correlation coefficient between the profiles. It is conducted by applying a normalized cross-correlation function and it requires approximation of cylindrical surfaces using the Legendre-Fourier method. As the example two sets of measurement data are employed: the first from the CMM and the second one from the traditional radial measuring instrument. The measuring data are compared by analyzing the values of selected cylindricity parameters and calculating the coefficient of correlation between profiles.

Keywords: cylindricity, form deviation, measurement, Legendre polynomials, Fourier series.

1. Introduction

In order to obtain reliable results of cylindricity, it is necessary to apply a suitable measuring strategy. The strategy should enable appropriate representation of the analyzed surface and appropriate density of measuring points [1]. An important criterion for selecting a measuring strategy is the assumed predominant harmonic component of roundness and straightness profiles [2, 3]. In practice, it is hardly possible to measure a workpiece surface using the theoretical minimum number of points defined in ISO 12180-2 [4]. This standard describes other measuring strategies as well. The strategies provide specific information on a workpiece, yet their application is limited, as they do not make it possible to evaluate the entire cylindrical surface. ISO 12180-2 describes four limited measuring strategies: the strategy for measurement of roundness profiles, the strategy for measurement of generatrix lines, the “bird-cage” strategy (which is a combination of measurement of roundness profiles and generatrix lines), and the point strategy. These measuring strategies are shown in Fig. 1.

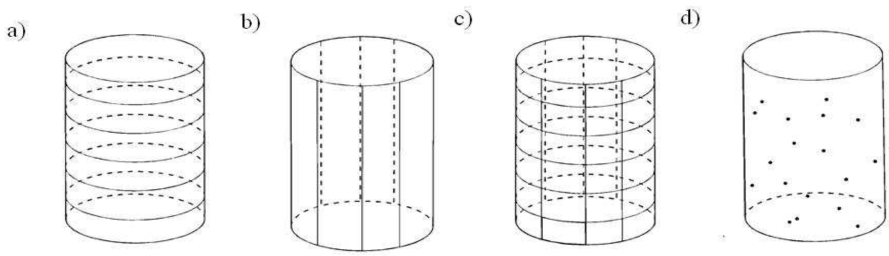

Fig. 1. Measuring strategies described by ISO 12180-1 [4]: a) the strategy for measurement of roundness profiles; b) the strategy for measurement of generatrix lines; c) the “bird-cage” strategy; d) the points strategy.

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In this study, cylindrical surfaces are approximated by combining the Fourier series and the Legendre orthogonal polynomials. This type of approximation can be employed to compare cylindricity profiles measured with different strategies.

2. Mathematical model of the method

By expanding the profiles in a Fourier series and approximating the expansion coefficients by means of the Legendre polynomials, we can write each cylindricity profile in the following form:

$$R(\varphi, z) \cong \sum_{m=0}^{M_z} L_m(z)\left(\frac{1}{2}a_{m0} + \sum_{n=1}^{M_\varphi}\left(a_{mn}\cos(n\varphi) + b_{mn}\sin(n\varphi)\right)\right), \tag{1}$$

$$L_m(z) = \overline{L}_m\left(\frac{2z}{H} - 1\right), \tag{2}$$

where:
- $\overline{L}_m$ – m-degree Legendre polynomial;
- φ – polar angle in the workpiece related cylindrical coordinate system;
- z – displacement along the Z-axis in the workpiece related cylindrical coordinate system;
- n – number of the profile harmonic component;
- M_z – degree of the Legendre approximating polynomial;
- M_φ – number of Fourier components being considered;
- a_{mn}, b_{mn} – coefficients of the profile approximation using the Legendre-Fourier method;
- H – cylinder height.

To compare cylindricity profiles measured with different strategies, it is necessary to perform an approximation using relationships (1–2). Assume that the approximation of the cylindrical surfaces by means of the Legendre-Fourier method with appropriate values of the M_z and M_φ degrees is based on the measuring points. The approximation coefficients determined with the first method are denoted by r in the upper index, while those determined with the other method by e in the upper index. In Ref. [5] the following form of a normalized cross-correlation function is proposed:

$$r(\tau) = \frac{2\sum_{k=1}^{N} R_r(\varphi_k + \tau, z_k) R_e(\varphi_k, z_k)}{\sum_{k=1}^{N} R_r^2(\varphi_k, z_k) + \sum_{k=1}^{N} R_e^2(\varphi_k, z_k)}, \tag{3}$$

where:
- k – number of subsequent measuring point;
- N – number of all measuring points;
- τ – phase shift between compared profiles.

The function described by the formula (3) is the expansion of the function that was used when comparing roundness profiles, presented in [6] and developed on the fundamental of the form of the cross-correlation function that is applied in digital signal processing.

The coefficient of coincidence of the compared profiles can be defined as:

$$\rho = \max r(\tau). \tag{4}$$

The value of τ at which the maximum of the cross-correlation function occurs is considered to be the phase shift between profiles. This value can be used to graphically represent the measured cylindrical surfaces in one diagram. One should note that the value of the correlation coefficient calculated from formulas (3) and (4) is sometimes not sensitive to differences between compared profiles. Therefore, the authors recommend comparing the values of cylindricity deviations of the profiles and then the visual comparison of profiles by superimposing them. The value of the coefficient given by formulas (3) and (4) can be regarded as an additional measure of coincidence of the profiles, only. The most important property of formulas (3) and (4) is the ability to determine the phase shift between the profiles that can be used for graphical comparison of compared profiles.

By applying Parseval's theorem, which assumes the orthogonality of basis functions and the cross-correlation function, we get:

$$r(\tau) = A_0 + \sum_{n=1}^{M_\kappa} \left(A_n \cos n\tau + B_n \sin n\tau\right), \tag{5}$$

where:

$$A_0 = \frac{1}{S_r + S_e} \sum_{m=0}^{M_z} \frac{1}{2m+1} a_{rm0} a_{em0} \,, \tag{6}$$

$$A_n = \frac{1}{S_r + S_e} \sum_{m=0}^{M_z} \frac{1}{2m+1} \left(a_{rmn} a_{emn} + b_{rmn} b_{emn}\right), \tag{7}$$

$$B_n = \frac{1}{S_r + S_e} \sum_{m=0}^{M_z} \frac{1}{2m+1} \left(a_{rmn} b_{emn} - a_{rmn} b_{emn}\right), \tag{8}$$

$$S_r = \sum_{m=0}^{M_z} \frac{1}{2m+1} \left(\left(a_{rm0}\right)^2 + \sum_{n=1}^{M_\varphi} \left(\left(a_{rmn}\right)^2 + \left(b_{rmn}\right)^2 \right) \right), \tag{9}$$

$$S_e = \sum_{m=0}^{M_z} \frac{1}{2m+1} \left(\left(a_{em0}\right)^2 + \sum_{n=1}^{M_\varphi} \left(\left(a_{emn}\right)^2 + \left(b_{emn}\right)^2 \right) \right). \tag{10}$$

$$M_\kappa = M_z M_\varphi . \tag{11}$$

Values denoted by the upper index *r* refer to the measurement data obtained by the radial method and by the index *e* to the measurement data obtained from the coordinate measuring machine Eclipse.

3. Experiment

The above mentioned concept of applying different measurement strategies was used in practice to compare form profiles of a cylindrical element. The dimensions of the element were the following: diameter = 40 mm, height = 100 mm. The element was manufactured by the NSK in the framework of a research grant concerning new methods of cylindricity measurements. The form and approximate values of cylindricity deviations were designed in the way that would make it easier to compare results taken from different measuring instruments.

Measurements were conducted using a ZEISS Eclipse 550 equipped with a triggering probe head and a computerized Taylor Hobson Talycenta machine for measurement of cylindricity. In both cases, cylindricity was measured in various cross-sections. Due to special properties of Legendre polynomials, only the first and the last cross-section in both measurements should be the same.

While employing the Hobson Taylor Talycenta, it was possible to collect 1024 measuring points per cross-section. When employing the Eclipse, 64 measuring points were sampled. The measurement data was used to determine the following values of cylindricity deviation:

– CYL_t = 30.1 µm, when applying a Zeiss Eclipse 550;
– CYL_t = 27.6 µm, when applying a Taylor Hobson Talycenta.

In a traditional approach, the comparison of profiles involves analyzing the differences between selected cylindricity parameters and it does not require applying the function of cross-correlation. This analysis, however, contains considerable errors, particularly if uncertainty of measurement with a CMM is taken into account [7, 8]. For instance, the uncertainty of the Zeiss Eclipse 550 for a measurement in three axes is 2.9 + L/250 µm, and the difference between the values of cylindricity deviation is 2.5 µm. When comparing profiles based on numerical values of parameters only, one may draw erroneous conclusions. If the cross-correlation function – relationships (5–10) – is employed to assess profiles, no such drawback is observed. Fig. 2 shows a diagram of the cross-correlation function calculated for the compared profiles. The maximum value of the function being represented in diagram form in Fig. 1 is 0.99. The relationship assumes that this value refers also to the coincidence between profiles. This high value of the correlation coefficient testifies to high cross-correlation of profiles.

When calculating the correlation between the profiles following values were assumed:

– number of Fourier components being considered: $M_\varphi = 15$;
– degree of the Legendre approximating polynomial $M_z = 6$.

A value of $M_\varphi = 15$ was selected because in measurements of geometrical quantities it is assumed that when analyzing deviations of form, harmonic components 2–15 should be taken to account. The degree of the Legendre polynomial was selected to be equal to 6 on the basis of results of some preliminary measurements. These results showed that the straightness deviation of the generatrix of the cylinder is significantly small in relation to the deviations of the cross-sections. Therefore, authors assumed that $M_z = 6$ would allow sufficiently accurate approximation of the measured profile. Of course, in the case of different elements such value might not be correct.

Applying the cross-correlation function described by relationships (5–10) is advantageous because the phase shift τ is used to equalize the values of the compared profiles within a phase. In this way, the compared profiles can be plotted together in one diagram and assessed visually. In the analyzed example, the cross-correlation function assumes the maximum value for $\tau \cong 0{,}6$ rad. This value is then used to equalize the compared profiles within a phase. The profiles can then be analyzed in various cross-sections, which helps visually to assess the coincidence between them.

The diagram in Fig. 3 shows the difference between values of the compared profiles at individual points of the profile. These are the differences of local deviations of the profiles from the least squares cylinders. In order to provide a more complete view of distribution of these differences, the diagram was plotted as a surface instead of dots.

Analyzing the differences shown in Fig. 3 one can notice that in some points they are quite large. There are a few possible sources of such differences. The first is that there is a large difference between the uncertainty of measurement by the radial method (for the instrument used in experiments it was about 0.1 µm) and by the applied coordinate measuring machine Eclipse (2.9 + L/250 µm in three axes). Another source of errors can be the difference of number of sampling points in measurements by the radial instrument (1024 points in one cross-section) and by the coordinate measuring machine (64 points in one cross-section). So, in case there are significant local irregularities of the profile, they may not be detected when only 64 points in the cross-section are sampled and detected in case when 1024 points are sampled. Another source of the difference can be, for example, dirt on the measured surface. Although, such reason is not very probable, because the operator did his best to keep the measurement conditions correct.

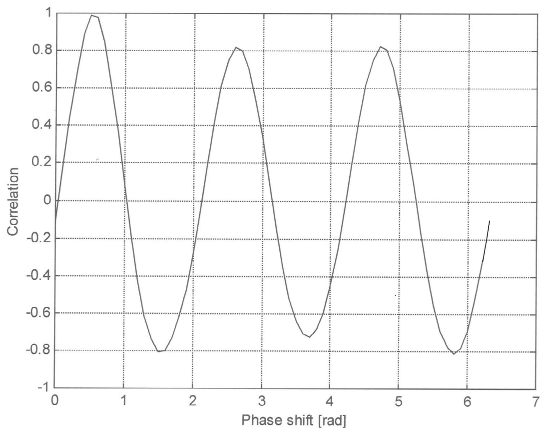

Fig. 2. Values of the cross-correlation function for the compared profiles.

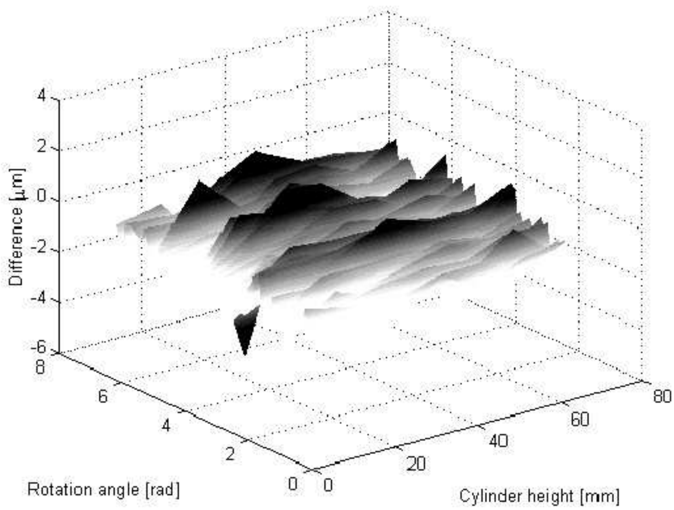

Fig. 3. Difference between the profile values.

Observing the profile changes in each cross-section, it seems particularly convenient to determine the difference between profiles at selected cross-sections of the element, as shown in Fig. 4.

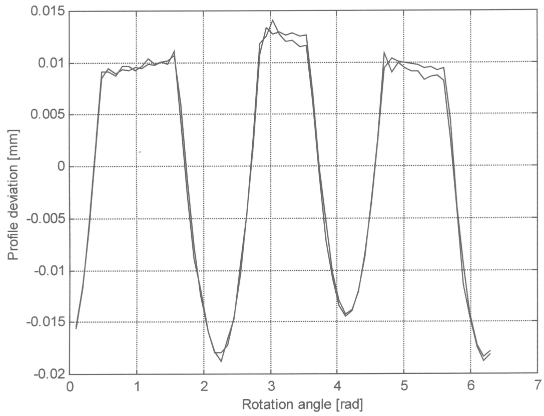

Fig. 4. Visual comparison of the profiles at the selected cross-section of the cylinder.

4. Summary

The experimental data confirm that the presented method makes it possible to compare cylindricity profiles measured with different instruments and strategies. The quantitative study involves calculating the correlation coefficient between profiles, while the qualitative analysis requires visual comparison of profiles. The results also show that the correlation coefficient provides more information about profile similarity than the values of selected parameters of cylindricity, which might be confusing. By taking account of the phase shift in relationship (5), it is possible to equalize and visually compare the analyzed profiles. It should be noted that of disadvantage is the fact that the coordinates of the measuring points need to be known, which is not always possible, especially in the case of special-purpose systems for measurement of cylindricity deviations. However, the value of the correlation coefficient calculated from the formulas (3) and (4) is sometimes not sensitive to differences between compared profiles. Therefore, the authors recommend comparing the values of cylindricity deviations of the profiles and then the visual comparison of profiles by superimposing them. The value of the coefficient may be regarded as an additional measure of coincidence of the profiles only. Despite this limitation, the proposed concept can be used to assess the accuracy of measurement of cylindricity deviations on CMMs, as it helps to perform a qualitative and quantitative analysis of profiles measured with different instruments and strategies.

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