BROADBAND MICROWAVE CORRELATOR OF NOISE SIGNALS

Waldemar Susek, Bronisław Stec

Military University of Technology, Faculty of Electronics, Institute of Radioelectronics, Kaliskiego 2, 00-908 Warsaw, Poland
(✉ Waldemar.Susek@wel.wat.edu.pl, +48 22 683 9831, Bronislaw.Stec@wel.wat.edu.pl)

Abstract

A real narrowband noise signal representation in the form of an analytical signal in the Hilbert space is presented in the paper. This analytical signal is illustrated in a variable complex plane as a mark with defined amplitude, phase, pulsation and instantaneous frequency. A block diagram of a broadband product detector in a quadrature system is presented. Measurement results of an autocorrelation function of a noise signal are shown and the application of such solution in a noise radar for precise determination of distance changes as well as velocities of these changes are also presented. Conclusions and future plans for applications of the presented detection technique in broadband noise radars bring the paper to an end.

Keywords: noise signal, correlation receiver, Hilbert space.

1. Introduction

An analytical signal is a kind of a complex representation of a real signal [1]. A complex signal $z(t)$ is called an analytical signal of a signal $x(t)$ such that:

$$z(t) = x(t) + jy(t). \tag{1}$$

As results from the definition, the real part of the complex signal $z(t)$ is the signal $x(t)$ itself while the imaginary part is the Hilbert transform [2] of the signal $x(t)$:

$$y(t) = H\{x(t)\} = \frac{1}{\pi} \int_{-\infty}^{+\infty} \frac{x(\tau)}{t - \tau} d\tau. \tag{2}$$

An analytical representation (1) can be attributed to all signals for which the integral (2) is concurrent. If the analytical signal is known, the real signal can be easily found from (1) as:

$$x(t) = \text{Re}\{z(t)\}. \tag{3}$$

On the other hand, if the Hilbert transform $y(t)$ is known, the original signal $x(t)$ can be determined after calculation of the reverse Hilbert transform:

$$x(t) = H^{-1}\{y(t)\} = -\frac{1}{\pi} \int_{-\infty}^{+\infty} \frac{y(\tau)}{t - \tau} d\tau. \tag{4}$$

The imaginary part of an analytical signal is uniquely related to the Hilbert transform which has the essential weight in the theory of complex description of signals. An analytical signal can be presented in a variable complex plane as shown in Fig. 1.
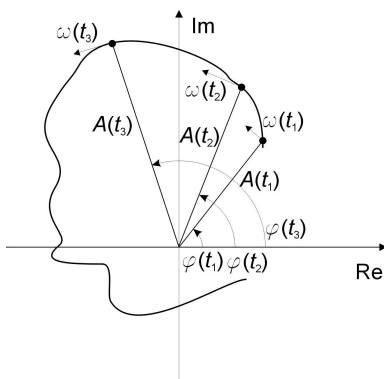
Fig. 1. Mark in a variable complex plane representing an analytical signal.

Parameters of the mark presented in Fig.1 are functions of time, so the position and length of the mark change in time also while the end of the mark draws an adequate trajectory. On the basis of the above-defined notion of the mark, the following concepts of an analytical signal can be defined [3]: instantaneous amplitude, instantaneous phase, and instantaneous frequency. The instantaneous amplitude of the analytical signal is the length of its mark equal to the signal module, *i.e.*:

$$A(t) = |z(t)| = \sqrt{x^2(t) + y^2(t)}. \tag{5}$$

The instantaneous phase of the analytical signal is described as the argument of a complex function $z(t)$ or as the imaginary part of the logarithm $z(t) \neq 0$ of a complex function of the real variable t:

$$\phi(t) = \arg\{z(t)\} \equiv \mathrm{Im}\{\ln(z(t))\} \quad [\mathrm{rad}]. \tag{6}$$

On the other hand, the instantaneous angular velocity of the mark rotation is called the instantaneous pulsation of the analytical signal, *i.e.*:

$$\omega(t) = \frac{\mathrm{d}\phi(t)}{\mathrm{d}t} \quad [\mathrm{rad/s}]. \tag{7}$$

The analytical signal, shifted in frequency, of a real narrowband noise signal can be defined in the following way:

$$z_N(t) = [X(t) + \mathrm{j}Y(t)]\exp(\mathrm{j}\omega_0 t), \tag{8}$$

where $X(t)$ and $Y(t)$ are the independent stationary random processes of Gaussian distribution with mean values equal zero and ω_0 is the median angular frequency of the band occupied by the noise signal. Therefore, according to dependence (3) the narrowband noise signal $s_N(t)$ in Hilbert space is described as [4, 5]:

$$s_N(t) = \mathrm{Re}\{z_N(t)\} = X(t)\cos(\omega_0 t) - Y(t)\sin(\omega_0 t). \tag{9}$$

The dependence (9) is used in the description of noise signals in next parts of the paper.

2. Direct correlation detection of the noise signal

The correlation-type receiver is a typical element, among others, entering into the noise radar. Noise radars are the radars which use random or pseudo-random signals for target illumination. Their basic parameters are a broad band of the signal, low power density and capability to obtain high sensitivity of receiving devices using non-conventional methods [6]. In particular, the principle of correlation coherent detection of the noise signal allows the operation of many devices in the same frequency range without mutual interference. The idea of direct correlation detection of the noise signal transmitted and received by the noise radar is presented in this chapter. The analysis of detector operation was carried on the basis of the schematic diagram shown in Fig. 2.
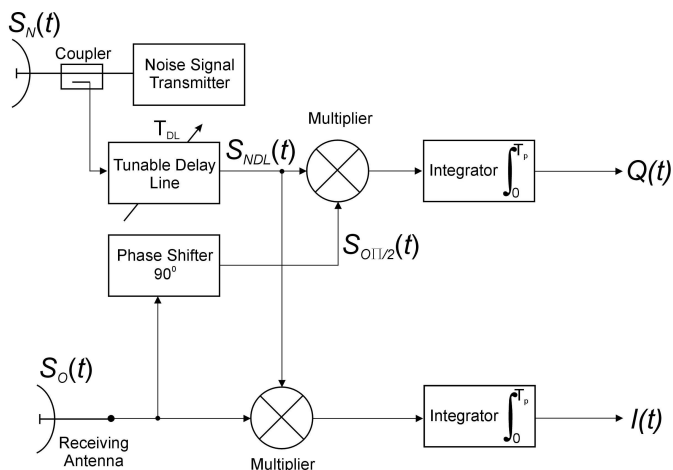

Fig. 2. Schematic diagram of quadrature detection of the noise signal in the correlation detector.

Let us assume that the transmitter generates a signal in the form of narrowband noise of Gaussian distribution with mean value equal zero and a variance equal σ^2. On the basis of (9), signals at the individual points of the system can be described in the time domain as follows:

$$S_N(t) = C_0 \left[X(t)\cos(\omega_0 t) - Y(t)\sin(\omega_0 t) \right], \tag{10}$$

$$S_{NDL}(t) = C_1 \left[X(t-T_{DL})\cos(\omega_0(t-T_{DL})) - Y(t-T_{DL})\sin(\omega_0(t-T_{DL})) \right], \tag{11}$$

$$S_{O\Pi/2}(t) = C_2 \left[X(t-T)\cos\left(\omega_0\left(t-\frac{2R'}{c}\right)-\frac{\pi}{2}\right) - Y(t-T)\sin\left(\omega_0\left(t-\frac{2R'}{c}\right)-\frac{\pi}{2}\right) \right], \tag{12}$$

$$S_O(t) = C_3 \left[X(t-T)\cos\left(\omega_0\left(t-\frac{2R'}{c}\right)\right) - Y(t-T)\sin\left(\omega_0\left(t-\frac{2R'}{c}\right)\right) \right], \tag{13}$$

where *R'* is the instantaneous distance between the radar and the object, T_{DL} is the time delay in the delay line, ω_0 is the median frequency of the band occupied by the noise signal and C_n are the constant coefficients.

The relation describing the instantaneous distance has the form:

$$R' = \frac{cT}{2} + vt + D_r \cos(\omega_r t). \tag{14}$$

In relation (14) T is the signal delay time on the radar-object-radar path, v is the radial velocity of the object, while D_r i ω_r are the amplitude and the pulsation of the additional harmonic movement of certain object parts, respectively. The object as a whole can move with radial velocity v or stay in rest.

In the result of multiplication of signals $S_{NDL}(t)$ and $S_O(t)$, $S_{OP/2}(t)$ and $S_{NDL}(t)$ and after integration of the obtained products, the two quadrature output signals $I(t)$ and $Q(t)$ can be expressed as:

$$I(t) = C_4 A \cos\left(\omega_0(T - T_{DL}) + \frac{2\omega_0 v}{c}t + \frac{2\omega_0 D_r}{c}\cos(\omega_r t)\right)$$

$$I(t) \overset{T=T_{DL}}{=} C_4 \sigma^2 \cos\left(\frac{2\omega_0 v}{c}t + \frac{2\omega_0 D_r}{c}\cos(\omega_r t)\right) \tag{15}$$

$$Q(t) = C_4 A \sin\left(\omega_0(T - T_{DL}) + \frac{2\omega_0 v}{c}t + \frac{2\omega_0 D_r}{c}\cos(\omega_r t)\right)$$

$$Q(t) \overset{T=T_{DL}}{=} C_4 \sigma^2 \sin\left(\frac{2\omega_0 v}{c}t + \frac{2\omega_0 D_r}{c}\cos(\omega_r t)\right) \tag{16}$$

Relations (15) and (16) describe quadrature components of the correlation function of the noise with limited bandwidth. A complex representation of the correlation function of the narrowband noise signal can be formed in this case as:

$$R((T - T_{DL}), t) = I(t) + jQ(t). \tag{17}$$

It results from the presented analysis that in the equipment built according to the schematic diagram presented in Fig. 2, in contrary to digital processing [8], the correlation function of the narrowband noise signal is determined. If $R((T-T_{DL}), t)$ is the complex representation of the correlation function of the narrowband noise signal, then the instantaneous amplitude, phase, and pulsation, similarly to (5, 6), and (7), amount to, respectively:

$$A = \frac{1}{T_p}\int_0^{T_p}[X(t-T)X(t-T_{DL}) + Y(t-T)Y(t-T_{DL})]\,dt, \tag{18}$$

$$\varphi(t) = \arctan\left(\frac{Q(t)}{I(t)}\right), \tag{19}$$

$$\omega(t) = \frac{d}{dt}\left(\arctan\left(\frac{Q(t)}{I(t)}\right)\right), \tag{20}$$

where T_p is the integration time.

The relation (20) is of particular meaning in the presented mode of noise signal detection. It shows that it is possible to detect insignificant movements of an object with respect to the observational noise radar. The more the dynamic changes of the instantaneous phase the better precision of the object movement detection.

3. Broadband microwave quadrature detector

The broadband microwave quadrature detector is a modified six-port measurement module. The system consists of one power divider, one coupler, and two diode rings.
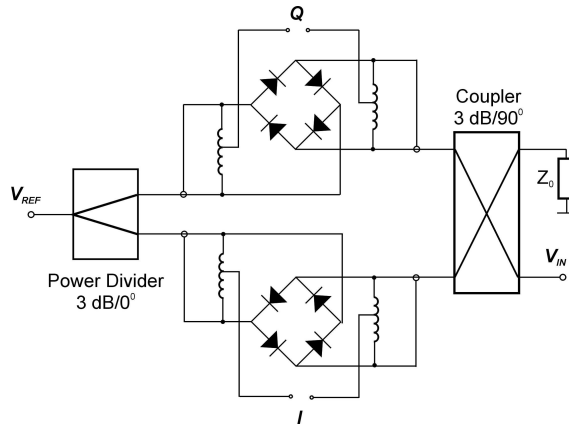

Fig. 3. Microwave quadrature detector with diode rings.

Input voltages V_{REF} and V_{IN} are divided into two equal parts and delivered to diode rings with adequate phase shifts. Diode rings play the role of multiplication systems.

Fig. 4. View of the upper side of the planar structure of the microwave multiplication system with transformers.

Fig. 5. View of the bottom side of the planar structure of the microwave multiplication system with transformers.

The mode of realization of the microwave multiplication system is shown in Figs 4 and 5. The diode ring is connected to a double-conductor line which changes over to an asymmetric strip line at the inputs of the V_{REF} and V_{IN} signals. Such transitions, in the microwave technique, realize the functions of transformers shown in Fig. 3. The suggested detector is a broadband microwave $I–Q$ detector. A detailed analysis of the system shown in Fig. 3 is presented in [7].

4. Measurement results

For random courses, contrary to expectation, the instantaneous values of these runs do not change in an arbitrary way, since there exists a certain relation between them resulting, among others, from inertia of real electronic circuits. For an estimation of this relation the autocorrelation function $R_x(t_1, t_2)$ [8, 9] of the process $X(t)$ is used which is the associative moment of the random variables $X(t_1)$ and $X(t_2)$:

$$R_x(t_1, t_2) = \mathrm{E}\big[X(t_1) X(t_2) \big] = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} x_1 x_2\, p(x_1, t_1; x_2, t_2)\, \mathrm{d}x_1 \mathrm{d}x_2. \tag{21}$$

The autocorrelation function of white noise at the output of an ideally rectangular band-pass filter with the bandwidth $B = f_2 – f_1$ and the median frequency $f_s = (f_2 + f_1)/2$ is described by dependence (22) and illustrated in Fig. 6.

$$\frac{R(\tau)}{R(0)} = \frac{1}{2\pi\tau B}\big[\sin(2\pi f_2 \tau) - \sin(2\pi f_1 \tau) \big]. \tag{22}$$

Experimental measurements of the autocorrelation function of a narrowband noise signal were carried out using a microwave quadrature detector. A block diagram of the measurement system is presented in Fig. 7.
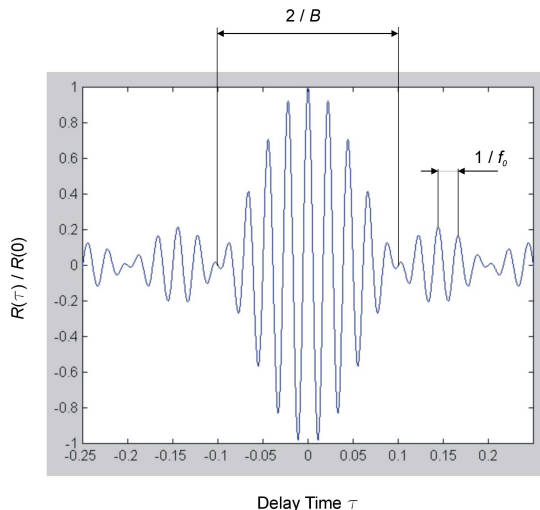

Fig. 6. Normalized autocorrelation function of white noise at the output of an ideally rectangular band-pass filter.

The microwave noise generator was constructed on the basis of an original semiconductor noise source and a sequence of amplifiers and microwave filters. Thereby a noise signal

source was achieved, operating at the median frequency $f_0 = 3$ GHz with a 3 dB bandwidth of $B = 480$ MHz and the spectral power density $G_N = -78$ dBm/Hz. The spectrum of the noise power was finally formed via output band-pass filter FPP with an uniformly flat transmittance profile.
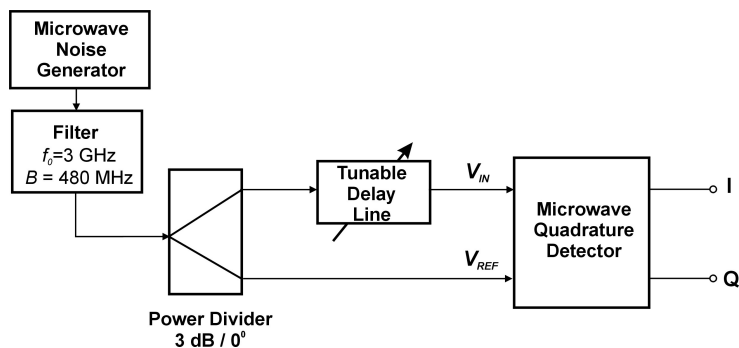

Fig. 7. Block diagram of the measurement system for autocorrelation.

A concentric line of controllable length was used in measurements. The applied microwave couplers and microwave power dividers operate in the $2 \div 4$ GHz band.
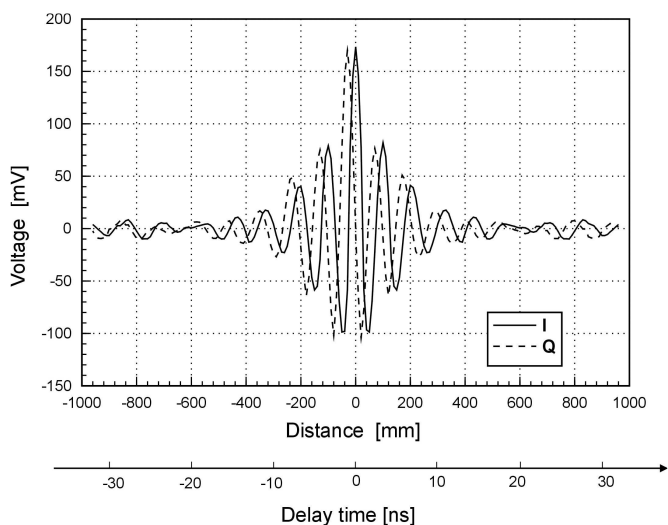

Fig. 8. Measured shape of the autocorrelation function of the narrowband noise signal.

The measurement result of the autocorrelation function of a narrowband noise signal in the form of two courses *I* and *Q* obtained at the output of the microwave quadrature detector is presented in Fig. 8. In the microwave range, experimental results confirm, with good approximation, the results of noise signal and its autocorrelation function calculations according to the mathematical model described earlier. For very high frequencies in the microwave range an analog system of the correlation detector can be realized, since for these frequencies a digital realization of the autocorrelation function is not available.

5. Conclusions and future plans

The suggested correlation detection method of broadband noise signals allows the construction of radars with noise signals. Then appear the radars which ensure better use of frequency bands that now are missing. Radar devices can be built for the similar frequency range but without jamming devices already operating in this range. Noise radars are characterized by a "pin-type" undetermination function. Therefore, the distance and Doppler frequency can be precisely and unambiguously determined by means of these radars. Noise signals coming from other sources operating at the same frequency range will not be correlated with the specific signal of the noise radar, thus the values of correlation functions of these signals will be very low. The possibility appears to construct anti-collision radars for moving objects which will not jam each other even for a huge number of such devices in a specified area. The frequency range of noise radars covers an interval from MHz to hundreds of GHz and this is only a technological limitation at the current development stage of electronics. Main application fields of noise radars are as follows: anti-collision radars[10], protection of objects, detection of motion/movement [11, 12], recognition and penetration of inaccessible objects, penetration of objects hidden in shallow soil layers, or detection of living beings in inaccessible areas. The authors' future plans will focus on application of the detector systems presented in the paper to selected noise radars.

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