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RESEARCH ON SPATIAL INTERRELATIONS OF GEOMETRIC DEVIATIONS DETERMINED IN COORDINATE MEASUREMENTS OF FREE-FORM SURFACES

Małgorzata Poniatowska

Bialystok Technical University, Chair of Materials Engineering and Mechanical Technology, Wiejska 45C, 15-351 Białystok, Poland (✉ mponiat@pb.edu.pl, +48 85 746 9261*)*

Abstract

Coordinate measurements are the source of digital data in the form of coordinates of the measurement points of a discrete distribution on the measured surface. The geometric deviations of free-form surfaces are determined (at each point) as normal deviations of these points from the nominal surface (the CAD model). Different sources of errors in the manufacturing process result in deviations of different character, deterministic and random. The contribution of random phenomena on the surface depends on the type of processing. The article suggests an innovation in applying the methods of analysis of spatial data in research on the geometric deviations randomness of free-form surfaces, consisting in testing their spatial autocorrelation. In the research on spatial autocorrelation of free-form surface deviations, Moran's *I* statistic was applied.

Keywords: coordinate measurements, free-form surface, geometric deviations, spatial autocorrelation.

1. Introduction

The coordinate measurement technique consists in determining the coordinate values of measurement points located on the surface of an object. As a result of the measurement, a set of discrete data is obtained in the form of the coordinates of the measurement points. From the point of view of CAD/CAM techniques, the most important feature of coordinate measurement is providing data concerning the object's geometry in a digital form.

In coordinate measurements of standard machine parts described by the use of simple geometric shapes, the macroinstructions built in the software are used. On the basis of the coordinates of the measurement points, first the associated geometric elements, and later their dimensions as well as the shape and location deviations are determined. The accuracy inspection is limited to comparing the determined dimensions with the data included in the design drawings.

In order to ensure functionality, ergonomics and aesthetics of products, machine parts composed of free-form 3D surfaces are often designed. Such parts are shaped by surfaces which cannot be described with simple mathematical equations. In designing, producing and measuring free-form surfaces, CAD/CAM techniques are applied. The accuracy inspection consists in digitalizing the workpiece under research (coordinate measurement with the scanning method), followed by comparing the obtained coordinates of the measurement points with the CAD design (model). The values of geometric deviations of the free-form surface, or normal deviations of measurement points from the nominal surface, can be calculated by previously determining the deviation components in the X, Y, Z directions [1, 2]. The software of coordinate measurement machines automatically performs such calculation for each measurement point in the UV scanning option. The processing accuracy inspection results may be presented in the form of a three-dimensional plot [3].

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Different sampling strategies (number and location of measurement points) provide different measurement results for the same surface. This is connected with the fact that measurement of a finite number of discrete points on the measured surface is actually described by an infinite number of points. Since geometric deviations are different at each point, measurement results depend on the number and location of these points [4, 5, 6]. Elkot [7] proposed to sample points from free-form surfaces based on the surface NURBS features.

There are generally two types of measurement data acquisition methods: contact measurement using a coordinate measuring machine (CMM) and non-contact measurement by using an optical/laser scanner. Numerically controlled CMMs (NC CMMs) equipped with ball-end touch trigger or scanning probes, are mainly used for workpiece validation in manufacturing.

Software for NC CMMs makes it possible to generate the path along which the probe stylus end moves lengthwise the actual surface on the basis of a CAD model. A typical solution is to measure some specified number of points with automatic probe radius compensation [8]. At the first stage of measurement, it is necessary to establish the relationship between the coordinate system of the model superimposed on the workpiece and that of the machine. To achieve this, the workpiece coordinate system is defined in the manual mode, and later the coordinate systems of the model and the workpiece are superimposed virtually. This common procedure makes it possible for the CMM software to generate theoretical measurement points on the workpiece (through the virtual model). Next, to obtain a more accurate mutual location of the workpiece and the model, after performing automatic scanning of a specified number of points (usually a few dozen points because of time limits), the obtained data should be fitted to the CAD model. The least square method provides an optimal solution, as demonstrated by Yau and Menq [9]. In this way the systematic error from the coordinate systems disagreement is removed. The matching accuracy increases with the number of measurement points. Geometric deviations of free-form surfaces are attributed to many phenomena that occur during the machining process, both deterministic and random in character. These phenomena with their consequent machining errors can be described in the space domain. In machining workpieces including free-form surfaces, multi-axis machining is applied. Different combinations of machining parameters may produce variations in the final product surface quality. Researchers have proposed correction and compensation or optimization techniques to improve machining accuracy. Among them Bohez [10] has proposed compensation methods for the systematic error of a five-axis machine. Cho [11] has applied an iterative computational approach to correct the tool path for machining errors reduction. Ye and Xiong [12] have constructed an objective function measuring the deviation between the machined surface and designed surface and solved the optimization problem. The goal is to maximize the similarity between the actual surface and the designed surface. As a matter of fact geometric deviations will always occur in spite of any efforts to eradicate them. Surface finish accuracy can be improved by eliminating the sources of deterministic deviations determined from the processing and/or correcting the processing program. Elimination of the sources of random deviations is not possible due to their unpredictable character. Many publications mention the problems of the relationships between processing parameters and surface geometric deviations and process optimisation.

In coordinate measurement of free-form surfaces, spatial data is obtained which provides information on the processing and on geometric deviations in the spatial aspect. Deterministic deviations are spatially correlated however, lack of spatial correlation indicates their spatial randomness. Calculating the values of geometric deviations solely does not provide much information, neither with regard to the surface properties nor to the course of the machining process. Deviations of random values may be spatially correlated which is reflected in their determined distribution on a surface and is indicative of the existence of a systematic source in the course of processing.

This article suggests an innovation in the form of applying the methods of analyzing spatial data to research on geometric deviations of free-form surfaces. These methods make it possible to quantitatively qualify the spatial interdependence of the given data. The reasons which made the application of these methods possible are the availability of spatial data and the introduction of new software in which spatial analysis is explicitly done. A basic concept used in modern spatial analysis is spatial autocorrelation. The literature states that Moran's *I* statistic is used in the majority of cases in order to test the existence of spatial autocorrelation in data. Cliff and Ord [13] justify that choice. Identifying spatial autocorrelation of geometric deviations proves the existence of a systematic, repetitive processing error. In such a case, the theoretical spatial modelling methods suggested by Cliff and Ord [13], as well as by Kopczewska [14], may be applied to fitting a surface regression model describing the deterministic deviations. In engineering practice, an advanced CAD software for surface modelling may be applied. The first step in model diagnosing is to examine the model residuals for the existence of spatial autocorrelation. Moran's *I* test is also used for this purpose.

The author presents the theoretical foundations along with a detailed plan of testing the spatial autocorrelation of measurement data. The described tests were carried out on a free-form surface obtained in the process of milling. The computations were made in the ***R-Gui*** programme which is a software environment for statistical computing and graphics.

2. Geometric deviations characterized using discrete measurement data

Geometric deviations are attributed to many factors. Different sources of errors in the manufacturing process leave traces on the surface, and geometric deviations are a cumulated effect of the influence of these sources. Deviations may be divided into three components: shape deviations, waviness and roughness. The components connected with the shape deviations and waviness are surface irregularities superimposed on the nominal surface, resulting in a smooth surface and they are most often deterministic in character. The component connected with random phenomena, including the surface roughness, is irregularity of high frequency. The actual surface is the effect of superimposition of the shape deviations, waviness and roughness on the nominal surface. The contribution of random phenomena on the surface depends on the type of processing The literature data indicate that after the finished milling process, values of random geometric deviations of the surface are greater than those of the deterministic deviations.

Shape deviations are caused, among others, by deviations of machine tool ways, deviations of the machine tool parts, and improper fixing. The surface waviness results from, among others, geometric deviations or tool movement deviations, and vibrations of the machine tool or the processing tool. Roughness is a result of the shape of the tool blade and the tool's longitudinal feed or in-feed as well as of vibrations at the workpiece-tool contact.

In coordinate measurements, the coordinates of a finite number of points on the surface of the workpiece are determined. The aim is to determine a smooth surface superimposed on the nominal surface. However, in the measurement process, the random component and the deterministic component overlap each other. In consequence, the spatial coordinates collected at each measurement point include two separate components. The component connected with the deterministic deviations represents the smooth surface trend and is spatially correlated. The random component, on the other hand, is weakly correlated and is considered to be of a spatially random character. A surface constructed on measurement points is therefore more complex than a nominal surface.

3. Research methods of spatial data autocorrelation

Spatial autocorrelation refers to systematic spatial changes. In general, positive autocorrelation means that the observed feature values in a selected area are more similar to the features of the contiguous areas than it would result from the random distribution of these values. In the case of negative spatial autocorrelation, the values in the contiguous areas are more differing than it would result from their random distribution. A lack of spatial autocorrelation means spatial randomness. The values observed in one area do not depend on the values observed in the contiguous areas, and the observed spatial pattern is as much probable as any other spatial pattern.

In order to test the existence of spatial dependence, global and local Moran's and Geary's statistics for a given variable are applied. The spatial effects range may be researched by means of analyzing the lag in the spatial process, and the structure of spatial dependence – by testing and selecting weighting matrices defined according to different criteria. The structure of weights was described by Cliff [13] and by Kopczewska [14].

3.1. Moran's I statistic

The literature data state that the Moran's *I* statistic is used in the majority of cases; it can be applied to analyzing spatial data of both normal and unknown (randomization) probability distribution [13, 14].

In adapting methods of spatial statistics, concerning research on spatial autocorrelation, to research on geometric deviations, the following needs to be determined:

- ε_i – geometric deviation at each measurement point,
- $\bar{\varepsilon}$ – arithmetic mean of geometric deviation at n – measurement points,
- w_{ij} – weighting coefficients, elements of weighting matrices reflecting spatial relations between ε_i and ε_j.

A spatial weighting matrix defines the structure of the spatial neighbourhood. The matrix measures spatial connections and is constructed in order to specify spatial dependence. One of the possible dependence structures is assumed, *e.g.* neighbourhood along a common border, neighbourhood within the adopted radius or within the inverse of distance. In research on geometric deviations, it is most suitable to make the spatial interrelations dependent on the distance between the measurement points, in particular on the inverse of the minimum straight-line distance.

As a result of scanning, the coordinates (as well as geometric deviations) of the points distributed on the surface along a regular grid are obtained. The distance between the *i*-th and *j*-th point, according to the Euclidean metric, is as follows:

$$d_{ij} = \left[\left(x_i - x_j \right)^2 + \left(y_i - y_j \right)^2 \right]^{\frac{1}{2}}, \tag{1}$$

where:

- x_i, y_i – *i*-th point coordinates,
- x_j, y_j – *j*-th point coordinates,
- d_{ij} – distance between the *i*-th and *j*-th measurement point.

If it is assumed that the dependence between the data values at the *i* and *j* points decreases when the distance increases, this relation can be described in the following way:

$$w_{ij} = d_{ij}^{-k}, \tag{2}$$

where:
- $w_{ij} = 0$ for $i = j$,
- k – constant ($k \geq 1$).

The spatial autocorrelation coefficient has the following form:

$$I = \frac{n}{S_0} \frac{\sum_{i=1}^{n}\sum_{j=1}^{n} w_{ij}\left(\varepsilon_i - \bar{\varepsilon}\right)\left(\varepsilon_j - \bar{\varepsilon}\right)}{\sum_{i=1}^{n}\left(\varepsilon_i - \bar{\varepsilon}\right)^2}, \tag{3}$$

where: $S_0 = \sum_{i=1}^{n}\sum_{j=1}^{n} w_{ij} \ \left(i \neq j\right)$.

Moran's I statistic has an asymptotically normal distribution (for $n \to \infty$).

The *Moran's I* statistic indicates whether there is a spatial effect of agglomeration or not. Positive and significant values of the statistics imply the existence of positive spatial autocorrelation, *i.e.* a similarity of observation in the specified distance d. Negative statistics values mean negative autocorrelation, *i.e.* diversification of the tested observations. The Moran*'s I* statistic is interpreted as the correlation coefficient, although its value is not limited to the [-1, 1] interval. Such correlation takes place between the variable value at location i and the variable values at the neighbouring locations. The spatial autocorrelation coefficient may be interpreted in a similar way as the linear correlation coefficient. In the case of linear correlation, the squared correlation coefficient is the approximate determination coefficient of a model. In examining the variables interdependence, the extent to which the information included in both variables is shared is also tested. If, for example, the linear correlation coefficient between two variables is 0.7, the variability of one variable explains the variability of the other in approx. 50%. If the correlation coefficient is 1, the information on one variable is sufficient to fully determine the other and no additional information is needed. In the case of spatial autocorrelation, the I coefficient value describes the interdependence between variables in space. If, for instance, the I coefficient = 0.7, the location explains the variability of a given observation in approx. 50%. Obtaining data from the neighbouring locations provides new information only partially [14].

3.2. Null hypothesis of no spatial autocorrelation verification

After having determined the coefficient I, the null hypothesis of no spatial autocorrelation at the assumed significance level needs to be verified, Upton and Fingleton showed examples in [15]. The distribution moments can be determined both at the assumption that the data (deviations) comes from the normal distribution population and at the assumption that it comes from the population of an unknown probability distribution. When the number of localities is large it is reasonable to use the normal approximation.

Assuming a normal probability distribution for geometric deviations, the E(I) expected value and the var(I) variance are calculated using the formulae [13, 15]:

$$\mathrm{E}\left(I\right) = \frac{-1}{n-1}, \tag{4}$$

$$\mathrm{var}(I) = \frac{\left(n^2 S_1 - nS_2 + 3S_0^2\right)}{\left(n-1\right)\left(n+1\right)S_0^2}, \tag{5}$$

where:

– $S_1 = \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\left(w_{ij} + w_{ji}\right)^2 \quad (i \neq j)$,

– $S_2 = \sum_{i=1}^{n}\left(w_{i(.)} + w_{(.)j}\right)^2$,

– $w_{i(.)} = \sum_j w_{ij}$, $w_{(.)j} = \sum_i w_{ji}$.

The expected value (4) of the Moran's *I* statistic approaches 0, which might be interpreted as randomness [13, 14, 15].

Verifying the hypothesis of no spatial autocorrelation (or randomness) in the data set under research follows the plan listed below:

1. formulating the H_0 null hypothesis: data is not spatially correlated. The alternative H_1 hypothesis: data is spatially correlated,
2. assuming the significance level or the probability of rejecting the null hypothesis when it is true,
3. calculating the test statistics (standard normal deviate) $z = I_p - E(I)/\sqrt{\mathrm{var}(I)}$, I_p – the coefficient evaluated from experimental sample (3), distribution moments calculated using the formulae (4) and (5),
4. determining the limit value of the *z* test statistics (Moran*'s I* standard deviate); for the adopted significance level $z(\alpha) = z_\alpha$, which means that if $z < z_\alpha$ there is no reason for rejecting the null hypothesis, and in that case the null hypothesis is accepted, otherwise the alternative hypothesis is accepted.

In tests on geometric deviations, accepting the null hypothesis means that the tested deviations samples are spatially random.

4. Experimental investigations

The experiments were performed on a free-form surface of a workpiece made of aluminium alloy with the base measuring 120x100 mm (Fig. 1), obtained in the milling process using ball-end mill 10 mm in diameter, rotational speed equal to 7000 rev/min, working feed 600 mm/min and zig-zag cutting path in the XY plane. The measurements were carried out on a Mistral Standard 070705 Brown&Sharpe CMM (PC-DMIS software, MPE_E = 2,5 + L/250), equipped with a Renishaw TP200 touch trigger probe (3D form measurement deviation = ±1µm), a 20 mm stylus with a ball tip of 2 mm diameter.

Fig. 1. Model CAD of the surface.

4.1. Characteristics of the measured surface

The surface was scanned (applying automatic probe radius compensation) with the UV scanning option (the option built in PC-DMIS software), 750 uniformly distributed measurement points were scanned from the surface (30 rows and 25 columns), and the process of fitting the data to the nominal surface was then carried out in which the least square method was applied and all the measurement points were used. The measurement process was subsequently repeated, geometric deviations ε were computed.

The obtained measurement data are presented in a graphical form. Fig. 2 shows a spatial plot of the ε deviations with reference to the x and y nominal coordinates. The deviation distribution indicates that the measurement points contain both the deterministic and the random component (Figs 2, 3). The greatest deviation values appear in the lower part of the plot (Fig. 4), with y coordinates ranging from 10 mm to 20 mm. The peaks are located symmetrically with respect to the symmetry axis for the X direction. In connection with the surface geometry, the conclusion is reached that the deviations on this surface fragment are distributed in a deterministic way.

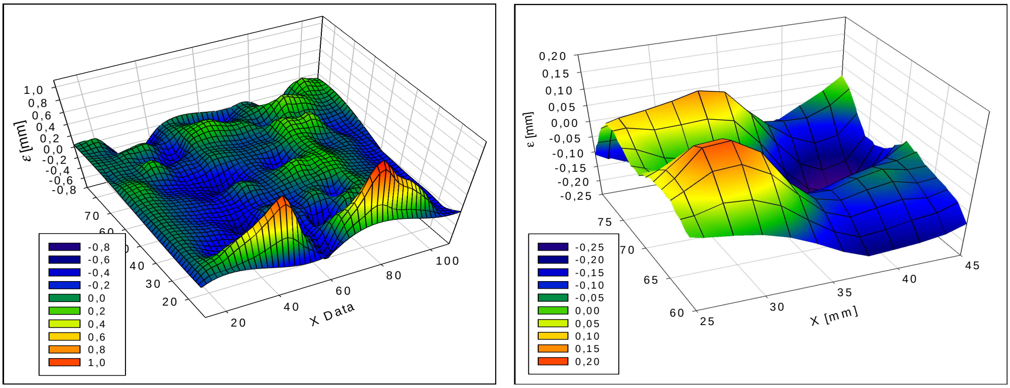


Fig. 2. Spatial plot of geometric deviations versus XY plane (left) and enlarged selected part of the plot (right).

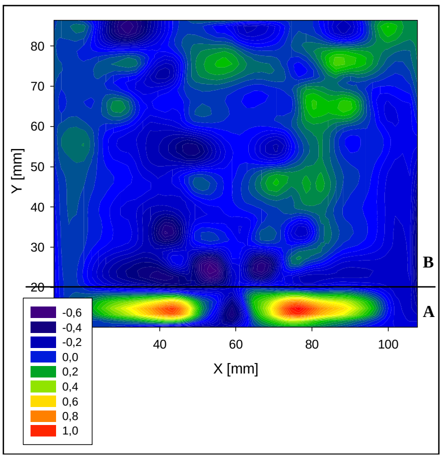
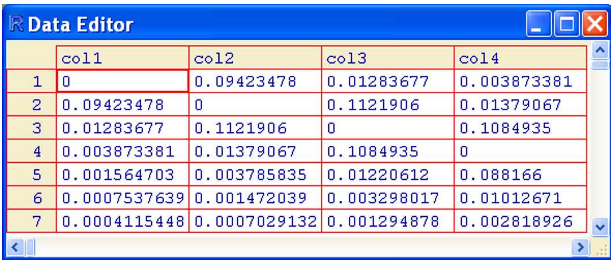


Fig. 3. Map of geometric deviations.

4.2. Tests on spatial autocorrelation of deviations

Tests on spatial autocorrelation of geometric deviations were subsequently carried out. The relationships between the deviations were made dependent on the reciprocal distances determined from formula (1). The elements of weight matrices, defining the dependencies between deviations at points i and j were calculated from formula (2), assuming the value of the constant as k = 3. A fragment of the weight matrix is shown in Fig. 4. Moving successively from the dependence (3) to (5), and later according to the items of the plan described in p. 3, the spatial autocorrelation coefficient was determined, and the null hypothesis on the lack of geometric deviations autocorrelation I was verified, assuming a normal plot approximation, with the significance level $\alpha = 0.05$ (the upper point of a standard normal distribution $z_\alpha = 1.645$). The computations were performed in the ***R-Gui*** programme. Fig. 5 presents the print screen image with the computation results.


Data Editor

	col1	col2	col3	col4
1	0	0.09423478	0.01283677	0.003873381
2	0.09423478	0	0.1121906	0.01379067
3	0.01283677	0.1121906	0	0.1084935
4	0.003873381	0.01379067	0.1084935	0
5	0.001564703	0.003785835	0.01220612	0.088166
6	0.0007537639	0.001472039	0.003298017	0.01012671
7	0.0004115448	0.0007029132	0.001294878	0.002818926

Fig. 4. The top left corner of the *W* matrix.

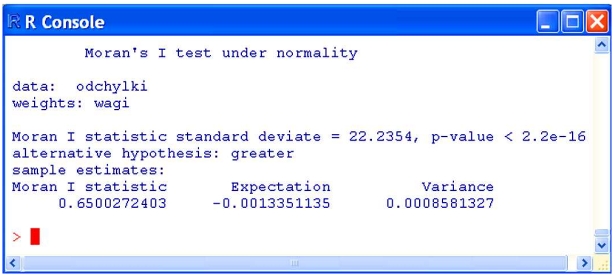


Fig. 5. Print screen image of *R-Gui* programme with computation results.

The null hypothesis of the lack of spatial autocorrelation was rejected (I = 0.650, z = 22.235, $z_\alpha = 1.645$, $z > z_\alpha$). The computation results show a clear positive autocorrelation of geometric deviations. In this case it is possible to predict in approx. 42% the values in the neighbouring points on the basis of the deviation value at any point. Considering that, increasing the number of measurement points (as opposed to the case with spatially independent values) does not provide much additional information because the deviation values can be partially predicted on the basis of the deviations at the neighbouring points.

In the next stage of the tests, the surface was divided into two areas which differed in respect of the character of deviations. It was assumed that in the area within y up to 20 mm – area ***A*** – deviations of a deterministic distribution prevailed, whereas in the area with y greater than 20 mm – area ***B*** – the distribution of deviations was random.

First, research was made on the spatial dependence between deviations in area ***B***, out of range of the deterministic errors. The plots of geometric deviations of this part of the surface are illustrated in Fig. 6. The same structure of spatial weight matrices as well as the same level of importance were assumed. The following results were obtained: I = 0.015, z = 0.533, $z < z_\alpha$. The null hypothesis was adopted. The deviations are indicative of the lack of spatial autocorrelation. In this surface fragment, the measurement points do not contain a deterministic component – the influence of the processing conditions was of a random character. The probability distribution of the deviation values is similar to the normal distribution (Fig. 7).

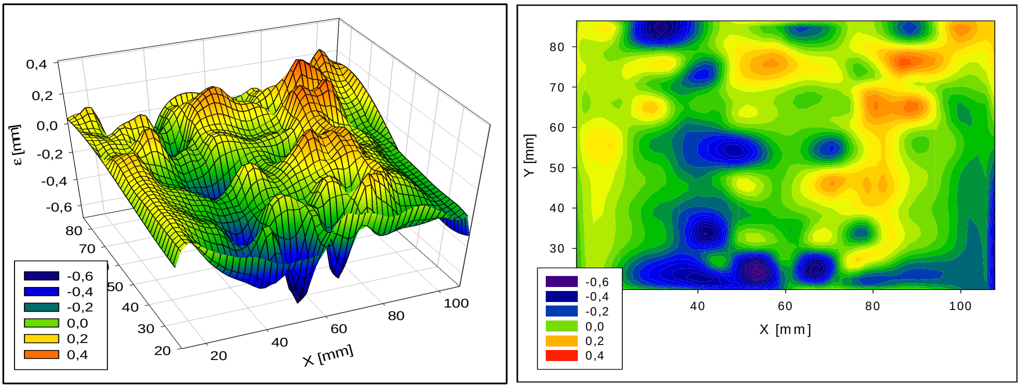


Fig. 6. Spatial plot and map of geometric deviations of the selected part of the surface versus *XY* plane.

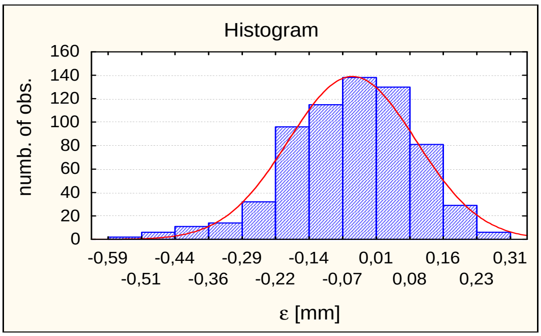


Fig. 7. Probability distribution of geometric deviations presented in Fig. 6.

In area ***A***, the results were as follows: I = 0.551, z = 10.058, $z > z_\alpha$. They are a sign that there is some significant spatial correlation between the geometric deviation values, which confirms the previously adopted suppositions. The test results indicate the existence of systematic processing errors. Further, the spatial model of deterministic geometric deviations needs to be determined and their sources of influence minimized, and/or the processing programme needs to be corrected.

5. Conclusions

Methods of spatial data analysis are most suitable in research on geometric deviations of free-form surfaces, because they allow to obtain information on spatial interdependence between the deviation values at individual measurement points. This is significant information concerning the accuracy of surfaces, both with regard to the surface properties and to the course of the machining process. These methods may be applied both to analyzing raw data, or data obtained directly from measurements, and also to researching residuals from surface regression models (trend surfaces) in tests of a models' adequacy. Detecting a positive spatial autocorrelation in geometric deviations is a proof that a systematic processing error has appeared, while the character of the error makes it possible to determine its value (spatial model) and later to eliminate the error by removing its source and/or by correcting the processing programme.

In the article tests on spatial autocorrelation of geometric deviations of a free-form surface were carried out and the Moran's *I* statistic was applied. The surface was divided into two areas, which differed in respect of the character of deviations. Tests were carried out separately for each area. Results of the tests confirmed the previously adopted suppositions. The Moran's *I* statistic made possible quantitative qualification of the spatial interdependence of geometric deviations.

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