MULTIPARAMETER APPROXIMATION OF TRANSDUCER TRANSFER FUNCTION BY KRIGING METHOD

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Abstract

The paper presents an application of the ordinary kriging method to predict multiparameter transfer function values in selected points in a transducer. This method allowed to soften the severity of measuring regime during determination of the transducer transfer function.

Keywords: correction of transducer transfer function, kriging.

1. Introduction

An approximation method for transducer multiparameter transfer function is widely addressed in literature [15, 16, 17]. Unfortunately the transducers' transfer function measurements with the use of this method are to meet severe measuring conditions, *i.e.* whilst measuring the relation between one of the input variables and the output, all other parameters influencing the output must remain stable. This requirement can be very difficult to meet. The best example of it is a flow meter transducer, as during measurements of its input vs. output signal dependence on the drop of pressure, the temperature and absolute pressure values have to be maintained at stable level. That is why it is necessary to develop mathematical methods allowing to soften these measuring conditions. The kriging method seems to be promising, as basing on values measured in points from a certain range, it allows to predict the values in other points from this range.

Kriging is a statistical method of estimating local variables if they are autocorrelative by nature. Such an autocorrelative structure can be expressed by semivariance of measured data. The method was named after D.G. Krige, an engineer working for the South African mining industry who first applied this method to estimate mineral reserves. Mathematically this method was explicated by the French mathematician G. Matheron [18]. The issue has been widely addressed in literature [1, 2-5, 12, 13, 14]. The method found its way to practicability in geostatic, hydrology, cartography and in many other domains [6-9, 14] and lately in image processing as it enables data inter- and extrapolation basing on a set of dispersed samples. Presently, there are three types of kriging distinguished: ordinary kriging, universal kriging and co-kriging.

In ordinary kriging the values in non-sampled points are estimated as the weighted mean of data obtained in sampled points and weights are determined from semivariance (Fig. 1.1).

The Fig.1 shows exemplary positions of measuring $z(x_1, y_1)$ ... $z(x_n, y_n)$ data points in three-dimensional space. The ordinary kriging method allows to estimate the z_p value in point x_p, y_p basing on known z_1, ... z_n values in points x_1, y_1, ... x_n, y_n and the geometrical distance from data points to the x_p, y_p point. The kriging mathematical basis was elaborated for three-dimensional space, because of its primary practical application. However the method can be

easily adopted to any multidimensional space. The algorithm does not change, only the number of computing steps increases.
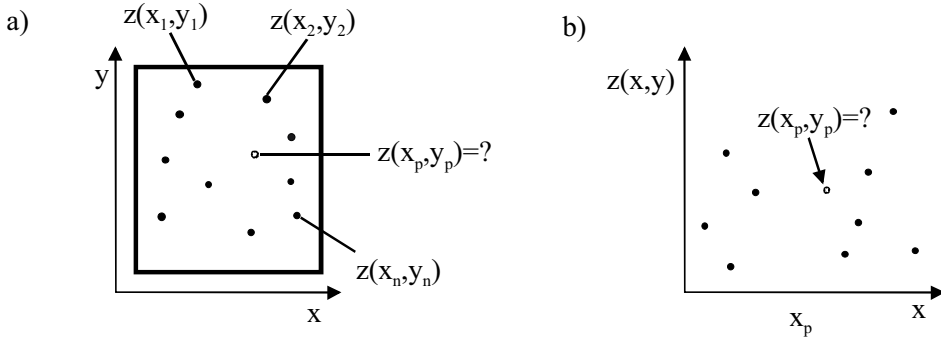

Fig. 1.1. Positions $z(x_i, y_i)$ of the measurement points and the predicted point $z(x_p, y_p)$:
a) x,y plane view; b) z,x, plane view.

Regardless of the kriging method used, the first computing step is to find a semivariance function, the one which expresses the degree of dependence between samples. After it is established, multipoint computing of to-be-predicted values can be carried out. When only one point is to be predicted the semivariance can be computed only for this point.

While estimating a transducer model it is assumed that it can be expressed mathematically, therefore the transfer function points are correlated. This correlation is to be found with semivariance.

2. Semivariance

Semivariance expresses the rate of dependence between particular points in a limited space. It is a measure of the spatial dependence between samples and expresses the behavior of a given variable in space or time.

To determine the semivariance function one should know the first two moments of statistic random functions ascribed to given phenomenon:

− the first moment (of averaging):

$$E[Z(x)] = m(x), \tag{2.1}$$

− the second one (of variance, covariance, semivariance):

$$var\{Z(x)\} = E\{[Z(x) - m(x)]^2\}. \tag{2.2}$$

If random variables $Z(x_1)$ $Z(x_2)$ have variance, they also have covariance, which is a function of position of x_1, x_2:

$$cov(x_1, x_2) = E\{[Z(x_1) - m(x_1)] \cdot [Z(x_2) - m(x_2)]\} = E\{Z(x_1) \cdot Z(x_2)\} - [m(x_1) \cdot m(x_2)]. \tag{2.3}$$

If $Z(.)$ is a stochastic process and is a function of spatial coordinate x, then semivariance is defined as:

$$\gamma(x_1, x_2) = \frac{1}{2} var\{Z(x_1) - Z(x_2)\}. \tag{2.4}$$

When the process $Z(.)$ is intrinsically stationary, the semivariance may be defined as:

$$\gamma(x_1, x_2) = \frac{1}{2} E\{[Z(x_1) - Z(x_2)]^2\}. \tag{2.5}$$

The experimental semivariance can be estimated for the N(h) paired observations, Z(x), Z(x+h), [18]:

$$\gamma(h) = \frac{1}{2P_h} \sum_{i=1}^{P_h} (Z(x_i) - Z(x_i + h))^2, \tag{2.6}$$

where:
- $Z(x_i)$ – sample value in point i,
- $Z(x_i+h)$ – sample value in distance h,
- P_h – number of sample pairs.

For models of transducers it can be assumed that Z(x) meets the requirements for a quadratic stationary state *i.e.* that the model is stationary and does not change its properties if the initial point in time or space is changed and what is more for every pair of $\{Z(x_i), Z(x_i+h)\}$ random variables there exists a covariance which depends only on the vector distance h.

For every value of vector h the difference $\{Z(x_i) - Z(x_i+h)\}$ has a finite variance and is x – independent. When the vector h value equals zero, the semivariance value is also zero. The semivariance is symmetrical with regard to h.

The value of semivariances between points depends on the distance between them. A small distance gives a small value and longer distance implies a greater value of semivariance. The graph of semivariance dependence on the distance h between points is called a semivariogram. The empirically obtained semivariograms are matched, as needed, with an adequate mathematical model:

linear:

$$\gamma(h) = c_0 + a \cdot h, \tag{2.7}$$

spatial:

$$\gamma(h) = \begin{cases} c_0 + c \cdot \left[1.5 \frac{|h|}{a} - 0.5 \left(\frac{|h|}{a} \right)^3 \right] & \text{for } |h| \le a \\ c_0 + c & \text{for } |h| > a \end{cases}, \tag{2.8}$$

exponential:

$$\gamma(h) = c_0 + c \cdot \left[1 - e^{-\frac{|h|}{a}} \right] \qquad \text{for } |h| > 0, \tag{2.9}$$

Gaussian:

$$\gamma(h) = c \cdot \left[1 - e^{-\frac{|h|^2}{a^2}} \right] \qquad \text{for } |h| > 0, \tag{2.10}$$

where:
- c_0 – initial value,
- c – range of semivariance (sill),
- k – slope of line,
- a – range of influence,
- h – magnitude of sampling interval.

Three characteristic parameters can be distinguished in a semivariograph: initial value c_0, for historical reasons called "a nugget", sill value and range of influence. The value c_0 expresses the variability of the examined quantity at a scale less than the interval of sampling

or can be originated due too low accuracy of measurement. The empirical semivariance for distance h=0 equals 0, as the point is compared with itself. The value after which the variance function does not increase anymore (approximately equal to the variance of sample) is called the sill value, and the distance from zero to the point at which the variance function attains 95% of its constant value is called the range of influence. This is also the longest distance at which the samples are correlated.

Depending on a particular need, the semivariance can be approximated with different functions or their modifications. In more complicated cases it would be a multinomial function or a spline.

3. Ordinary kriging

Ordinary kriging is the simplest way of predicting values in selected points. Assuming that the values of local variables are stationary, the values of dimensionless points are estimated basing on other dimensionless points.

Values are estimated using a linear function:

$$\hat{Z}(x_p) = \sum_{i=1}^{N} \omega_i Z(x_i) , \qquad (3.1)$$

where:
- N – number of measurement points,
- ω_I – weights,
- $Z(x_i)$ – measured values in points x_i.

The ω_I weights are called the kriging coefficients. Their values depend on sampling point positions and the position of the point whose value is to be estimated. The weights ascribed to samples should be selected to minimize the quadratic mean's error. This error is called the kriging variance σ_k^2 and is calculated for every sampling range and every configuration of the estimated range. The basic problem with estimator function (3.1) is to find the weights ω. These weights are determined from a system of equations, after taking into account the unbiased value of the estimator:

$$E\{\hat{Z}(x_p) - Z(x_p)\} = 0 \qquad (3.2)$$

and the condition of variance minimum:

$$\sigma^2(x_p) = \text{var}\{\hat{Z}(x_p) - Z(x_p)\} = \text{min.} \qquad (3.3)$$

From expressions (3.1) and (3.2), and applying normal distribution of sampled points we get:

$$E\{\hat{Z}(x_p) - Z(x_p)\} = \sum_{i=1}^{n} \omega_i E\{Z(x_i)\} - E\{Z(x_p)\} = m \sum_{i=1}^{N} \omega_i - m = 0, \qquad (3.4)$$

where m is an expected value.

For:

$$\sum_{i=1}^{n} \omega_i = 1 \qquad (3.5)$$

expression (3.3) equals zero and (3.3) can expressed as follows:

$$\sigma^2(Z(x_p)) = E\left\{\left[\sum_{i=1}^{n}\omega_i Z(x_i) - Z(x_p)\right]^2\right\}. \tag{3.6}$$

Expression (3.4) can be transformed by adding and subtracting the expected value m, which does not change the expression if (3.5) is met. After raising to the second power and transformation we get:

$$\sigma^2(Z(x_p)) = \sum_{i=1}^{n}\sum_{j=1}^{n}\omega_i\omega_j E\left\{(Z(x_i)-m)(Z(x_j)-m)\right\} - 2\sum_{i=1}^{n}\omega_i E\left\{(Z(x_i)-m)(Z(x_p)-m)\right\} +$$
$$+ E\left\{(Z(x_p)-m)^2\right\}. \tag{3.7}$$

The three expected values in expression (3.7) determine appropriate values of covariance and variance:

$E\left\{(Z(x_i)-m)(Z(x_j)-m)\right\}$ – covariance between two measuring samples,

$E\left\{(Z(x_i)-m)(Z(x_p)-m)\right\}$ – covariance between measuring sample and estimated value,

$E\left\{(Z(x_p)-m)\right\}$ – variance in estimated point.

Taking into account expressions (3.2-3.5) the (3.7) can be expressed regarding covariance and variance functions:

$$\sigma^2(Z(x_p)) = \sum_{i=1}^{n}\sum_{j=1}^{n}\omega_i\omega_j cov(Z(x_i),Z(x_j)) - 2\sum_{i=1}^{n}\omega_i cov(Z(x_i),Z(x_p)) + var(Z(x_p)). \tag{3.8}$$

In the next step, regarding expressions (2.5-2.8), covariance and variance functions expressed in distance h between particular points can be substituted to (3.8) giving:

$$\sigma^2(Z(x_p)) = \sum_{i=1}^{n}\sum_{j=1}^{n}\omega_i\omega_j cov(h_{i,j})) - 2\sum_{i=1}^{n}\omega_i cov(h_{i,p})) + cov(0), \tag{3.9}$$

where:
− $h_{i,j}$ – distance between points x_i and x_j,
− $h_{i,p}$ – distance between points x_i and x_p.

Successively, using (2.9), (3.9) can be expressed like:

$$\sigma^2(Z(x_p)) = 2\sum_{i=1}^{n}\omega_i\gamma(h_{i,p}) - \sum_{i=1}^{n}\sum_{j=1}^{n}\omega_i\omega_j\gamma(h_{i,j}). \tag{3.10}$$

The calculation of weights ω is carried out by establishing a minimum value of σ^2 variance. It can be achieved by using the Lagrange technique, which incorporates an additional coefficient μ called the Lagrange multiplier. That gives:

$$\sigma^2(Z(x_p)) - 2\mu\sum_{i=1}^{n}\omega_i. \tag{3.11}$$

Basing on the above, N partial differentials are generated and equated to zero:

$$\frac{\delta\left[\sigma^2(x_p) - 2\mu\sum_{i=1}^{n}\omega_i\right]}{\delta\omega_i} = 0. \tag{3.12}$$

After differentiation and ordering we get:

$$2\gamma\left(h_{i,p}\right) - 2\sum_{j=1}^{n}\omega_j\gamma(h_{i,j}) - 2\mu = 0. \tag{3.13}$$

After simplifying and rearranging of (3.13) and taking into account expression (2.5) we get N+1 equations with N+1 unknowns:

$$\sum_{i=1}^{n}\omega_j\gamma(h_{i,j}) + \mu = \gamma(h_{i,p}) \qquad j = 1,....,\, n$$

$$\sum_{i=1}^{n}\omega_i = 1, \tag{3.14}$$

which allows to compute weight coefficients ω and to determine the estimating function (3.1) at the same time as well as the value of variance:

$$\sigma^2(x_p) = \mu + \sum_{i=1}^{n}\omega_i\gamma(h_{i,p}). \tag{3.15}$$

The expanded first term in (3.14) will look as follows:

$$\begin{aligned}
\omega_1\gamma(h_{11}) + \omega_2\gamma(h_{12}) + ... + \omega_N\gamma(h_{1N}) + \mu &= \gamma(h_{1p}) \\
\omega_1\gamma(h_{21}) + \omega_2\gamma(h_{22}) + ... + \omega_N\gamma(h_{2N}) + \mu &= \gamma(h_{2p}) \\
\omega_1\gamma(h_{N1}) + \omega_2\gamma(h_{N2}) + ... + \omega_N\gamma(h_{NN}) + \mu &= \gamma(h_{Np}),
\end{aligned} \tag{3.16}$$

where:
− h_{ij} – distance between points x_i and x_j,
− h_{ip} – distance between points x_i and x_p,
  where the distances are established in a geometrical sense:

$$h = \left\| x_i\text{-}x_j \right\| = \sqrt{(x_{1i} - x_{1j})^2 + (x_{2i} - x_{2j})^2 + ... + (x_{ni} - x_{nj})^2}. \tag{3.17}$$

In matrix notation we get:

$$\begin{vmatrix} \gamma(h_{11}) & \gamma(h_{12}) & \gamma(h_{1n}) & 1 \\ \gamma(h_{21}) & \gamma(h_{22}) & \gamma(h_{2n}) & 1 \\ \gamma(h_{n1}) & \gamma(h_{n2}) & \gamma(h_{nn}) & 1 \\ 1 & 1 & 1 & 0 \end{vmatrix} \begin{vmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \\ \mu \end{vmatrix} = \begin{vmatrix} \gamma(h_{1,p}) \\ \gamma(h_{2,p}) \\ \gamma(h_{n,p}) \\ 1 \end{vmatrix}. \tag{3.18}$$

One should notice that the semivariance coefficients, which are positioned symmetrically to the main matrix diagonal have the same values, as they concern the same distances between the same points and the values at the main diagonal equal zero, as they refer to a single point. The zeroed values at the main diagonal can cause problems when the matrix is inversed. What is more, from the (3.16) system of equations it is clear that when the value of a new point is to be calculated its semivariance $\gamma(h_i,)$ has already been computed. However the semivariance $\gamma(h_{i,p})$ and the new values of weight coefficients are to be found. If the number of to-be-found points is significant, the number of calculations is also enormous. In matrix calculus the solution of the (3.18) system of equations is as follows:

$$\boldsymbol{\omega_p} = \boldsymbol{\Gamma_0^{-1}\gamma_p}, \tag{3.19}$$

where:

$$\boldsymbol{\omega}_p = \begin{vmatrix} \omega_1 \\ \omega_2 \\ \omega_n \\ \mu \end{vmatrix} \equiv \left(\frac{\boldsymbol{\omega}}{\mu} \right) \quad \boldsymbol{\Gamma}_0 = \begin{vmatrix} 0 & \gamma(h_{12}) & \gamma(h_{1n}) & 1 \\ \gamma(h_{21}) & 0 & \gamma(h_{2n}) & 1 \\ \gamma(h_{n1}) & \gamma(h_{n2}) & 0 & 1 \\ 1 & 1 & 1 & 0 \end{vmatrix} \equiv \left(\begin{matrix} \boldsymbol{\Gamma} & \mathbf{1} \\ \mathbf{1}^T & 0 \end{matrix} \right) \quad \boldsymbol{\gamma}_p = \begin{vmatrix} \gamma(h_{1p}) \\ \gamma(h_{2p}) \\ \gamma(h_{np}) \\ 1 \end{vmatrix} \equiv \left(\frac{\boldsymbol{\gamma}}{1} \right), \quad (3.20)$$

where:

$$\boldsymbol{\Gamma} = \begin{vmatrix} 0 & \gamma(h_{12}) & \gamma(h_{1n}) \\ \gamma(h_{21}) & 0 & \gamma(h_{2n}) \\ \gamma(h_{n1}) & \gamma(h_{n2}) & 0 \end{vmatrix}. \qquad (3.21)$$

Taking into account (3.15) and (3.16) the variance in predicting point x_p can be expressed by:

$$\boldsymbol{\sigma}^2(x_p) = \boldsymbol{\gamma}_p^{\ T} \, \boldsymbol{\omega}_p. \qquad (3.22)$$

An additional element inserted to those calculations is the input data standardization. It causes that the numerical range of values used in calculations is diminished which reduces the miscount errors. The input data standardization is carried out by calculating the mean value m and variance s for all coordinated measuring points and for values of samples in these points and then all the values are recalculated using the following formula:

$$f_i = \frac{(w_i - m)}{\sigma}, \qquad (3.23)$$

where:
- w_i – values of the given input data,
- m – mean value of input data,
- σ – variance.

The standardization procedure changes the range of independent variables to -2÷2, and values in particular points are contained in the range of 0÷1. What is more, the mean value of such recalculated input data equals 0 and the variance is 1. When calculations are finished, de-standardization is to be performed.

4. Transfer function approximation

In our case the ordinary kriging method described above was applied to scale the volume flow meter transducer described in [16, 17]. The physical complexity of flow problems makes it difficult to approach these processes from a mathematical standpoint. Hence, a large role is played by experimental research and mathematical modeling of flows [19, 20].

The magnitude of gas flow is calculated from the drop of gas pressure dP at the outlet of the orifice, with regard to gas temperature T and absolute pressure P. The temperature ranged from 263.15 K to 333.15 K (-10 ÷ +40 °C), absolute pressure from 100 kPa to 120 kPa and pressure drop at orifice was in the range of 0-250 Pa . The readings of measuring points are presented in Table 4.1.

Table 4.1. Sampled values for the transducer transfer function.

Lp.	Q [m³/h]	T [K]	dP [Pa]	P [Kpa]	Lp.	Q [m³/h]	T [K]	dP [Pa]	P [Kpa]
1.	0.2813	277.4161	49.0133	101	11.	0.0607	290.4066	5.3859	102
2.	0.6647	311.6259	201.7263	115	12.	0.0432	309.5759	3.6714	105
3.	0.4682	273.3354	117.1179	108	13.	0.4246	280.4239	97.4928	117
4	0.4398	304.1464	99.7551	106	14.	0.6233	283.7021	190.1696	120
5.	0.7338	305.8421	243.3591	118	15.	0.7472	278.9716	266.7299	116
6	0.2287	307.8978	34.4059	109	16.	0.1039	278.5927	10.8371	104
7.	0.6058	276.5274	183.5706	112	17.	0.4551	307.9217	105.1335	109
8.	0.6030	289.1413	177.2278	107	18.	0.3755	296.3382	77.2351	110
9	0.3044	283.5448	55.3498	113	19.	0.0095	295.1444	0.6906	100
10.	0.4543	305.1527	105.2518	116	20.	0.2697	278.9482	45.6644	112

Then, basing on the above data and using the ordinary kriging method, the flow values Q were calculated for selected (100, 105, 110, 115 and 120 kPa) values of absolute pressure P, temperature T (T = 273.15, 283.15, 293.15, 303.15 and 313.15 K) and differential pressure dP as in Table 4.1.

The first step was to calculate the approximating polynomials of flow Q dependence on differential pressure dP, with temperature T and pressure P as parameters:

$$Q_{P,T} = \sum_{i=0}^{3} \alpha_i dP^i. \tag{4.1}$$

The α_i coefficients of this polynomial for pressure P=100 kPa at sequential temperatures T are given in Table 4.2 and polynomial diagrams in Fig. 4.1.

Table 4.2. Values of α_i coefficients in dependence on temperature, for pressure P=100 kPa.

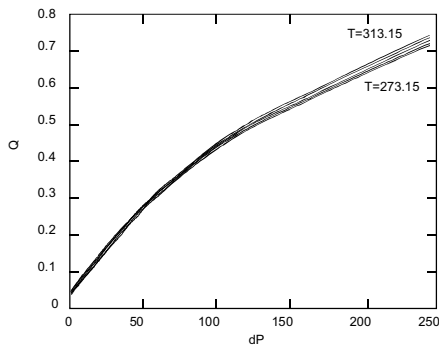| No.	T [K]	$\alpha_3 \cdot 10^7$	$\alpha_2 \cdot 10^4$	$\alpha_1 \cdot 10^3$	α_0
1.	273.15	0.3981	-0.223	5.983	0.0279
2.	283.15	0.4997	-0.228	6.047	0.0262
3.	293.15	0.4212	-0.225	6.103	0.0251
4.	303.15	0.4309	-0.228	6.159	0.0242
5.	313.15	0.4498	-0.230	6.195	0.0231

Fig. 4.1. Flow Q dependence on pressure dP with temperature T as parameter.

Approximation of α_i coefficients allowed to determine the approximating polynomials for two variables determining flow Q dependence on pressure dP and temperature T under P as a parameter.

$$Q = \sum_{i=0}^{3}\left(\sum_{j=0}^{2}\beta_{i,j}T^{j}\right)dP^{i}. \qquad (4.2)$$

The β coefficients of approximating polynomials are given in Table 4.3 and polynomial diagrams are in Fig. 4.2. with pressure P as a parameter.

Table 4.3. The β coefficients of approximating polynomial (4.2).

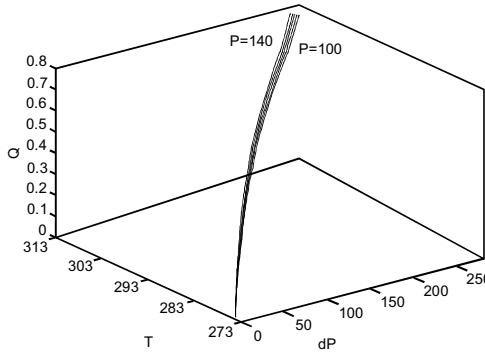| $\beta_{i,j}$	0	1	2
0	$7.25\cdot10^{-2}$	$2.19\cdot10^{-4}$	$2.09\cdot10^{-7}$
1	$1.98\cdot10^{-3}$	$2.16\cdot10^{-5}$	$2.99\cdot10^{-8}$
2	$5.32\cdot10^{-6}$	$-1.83\cdot10^{-7}$	$2.57\cdot10^{-10}$
3	$-3.15\cdot10^{-8}$	$4.55\cdot10^{-10}$	$-5.34\cdot10^{-13}$

Fig. 4.2. Flow Q dependence on temperature T with P as a parameter.

Approximation of β coefficients (4.2) allowed to determine the coefficients of the final approximating polynomial:

$$Q = \sum_{i=0}^{3}\left(\sum_{j=0}^{2}\left(\sum_{k=0}^{2}\gamma_{i,j,k}P^{k}\right)T^{j}\right)dP^{i}. \qquad (4.3)$$

They are listed in Table 4.4.

Table 4.4. Coefficients of polynomial approximation of flow transducer transfer function.

$\gamma_{i,j,k}$	2	1	0	$\gamma_{i,j,k}$	2	1	0
0,0	$7.21\cdot10^{-6}$	$1.21\cdot10^{-5}$	$8.06\cdot10^{-4}$	2,0	$8.09\cdot10^{-11}$	$5.42\cdot10^{-8}$	$6.16\cdot10^{-8}$
0,1	$3.10\cdot10^{-9}$	$1.89\cdot10^{-6}$	$1.32\cdot10^{-5}$	2,1	$1.12\cdot10^{-11}$	$-1.81\cdot10^{-9}$	$7.19\cdot10^{-9}$
0,2	$-4.38\cdot10^{-12}$	$2.03\cdot10^{-9}$	$1.39\cdot10^{-8}$	2,2	$4.17\cdot10^{-15}$	$2.05\cdot10^{-12}$	$2.18\cdot10^{-12}$
1,0	$8.21\cdot10^{-6}$	$1.90\cdot10^{-5}$	$5.03\cdot10^{-4}$	3,0	$2.59\cdot10^{-13}$	$-3.74\cdot10^{-10}$	$1.25\cdot10^{-9}$
1,1	$2.76\cdot10^{-10}$	$1.98\cdot10^{-7}$	$1.25\cdot10^{-6}$	3,1	$2.36\cdot10^{-15}$	$3.91\cdot10^{-12}$	$1.72\cdot10^{-11}$
1,2	$-1.52\cdot10^{-13}$	$2.76\cdot10^{-10}$	$1.21\cdot10^{-9}$	3,2	$8.07\cdot10^{-18}$	$-6.39\cdot10^{-15}$	$9.19\cdot10^{-15}$

In the next step the calculated coefficients of the polynomial were implemented along with the polynomial calculation algorithm (4.3) in a microcontroller which calculated the flow value basing on sampled data. Comparison of the measurement results with values taken from a reference device has shown that the error between the achieved polynomials in relation to the maximum flow did not exceed 1,21% in any case.

The above example shows that the ordinary kriging combined with the method of calculation of the polynomial approximating a multiparameter transducer transfer function can be an effective and cheap tool to determine transducers' model parameters. However, it was established that the number of input data for kriging is calculated by successive approximations. In the example above the calculations were primarily made for ten inputs. As the attempt to calculate the semivariance function rendered the result to be unsatisfactory, the number of inputs was increased to twenty. This proved to be satisfactory in terms of the obtained results. Calculations were also made for sixty inputs but the achieved improvement was not significant. It shows that for now there is no independent criterion allowing to establish the appropriate input number. It is possible that in more complex transducer transfer functions the necessity to either increase the number of samples or to regionalize the function *i.e.* divide the transducer range into sub ranges and approximate separately, will occur. The problem of selection of the method of sample points is a separate issue, not addressed in this paper.

References

[1] A. Arik: "Area Influence Kriging". *Mathematical Geology*, vol. 34, no. 7, October 2002, pp. 783-796.

[2] N. Cressie: "The origins of kriging". *Mathematical Geology,* vol. 20, no. 3, 1998, pp. 239-252.

[3] P. Goovaerts: "Ordinary Cokriging Revisited". *Mathematical Geology*, vol. 30, no. 1, 1998, pp. 21-42.

[4] T.C. Haas: "Kriging and automated semivariogram modeling within a moving window". *Atmospheric Environment,* 24A, 1990.

[5] V.R. Joseph: "Limit Kriging". *Technometrics*, vol. 48, no. 4, 2006, pp. 458-466.

[6] J.C. Jouhaud, P. Sagaut, B. Labeyrie : "A Kriging Approach for CFD/Wind Tunnel Data Comparison". *Journal of Fluids Engineering*, vol. 128, no. 4, 2006, pp. 847-855.

[7] Kai-Tai Fang, Hong Yin, Yi-Zeng Liang : "New Approach by Kriging Models to Problems in QSAR". *J. Chem. Inf. Comput. Sci*., vol. 44, no. 6, 2004, pp. 2106-2113.

[8] Lam K.Y., Wang Q.X., Hua Li: "A novel meshless approach – Local Kriging (LoKriging) method with two-dimensional structural analysis". *Computational Mechanics,* vol. 33, issue 3, 2004, pp. 235-244.

[9] L. Lebensztajn, C. Marretto, M. Costa, J.L. Coulomb: "Kriging: A Useful Tool for Electromagnetic Device Optimization". *IEEE Transactions on Magnetics*, vol. 40, no. 2, Mar. 2004, pp. 1196-1199.

[10] E. Pardo-Iguzquiza, P.A. Dowd: "Multiple indicator cokriging with application to optimal sampling for environmental monitoring". *Computers & Geosciences,* vol. 31, no. 1, 2005, pp. 1-13.

[11] M.L. Stein: *Interpolation of Spatial Data: Some Theory for Kriging. Series in Statistics.* Springer, New York, 1999.

[12] H. Subramanyam, S. Pandalai: "On the Equivalence of the Cokriging and Kriging Systems". *Mathematical Geology*, vol. 36, no. 4, May 2004, pp. 507-523.

[13] D.J.J. Walvoort, J.J de Gruijter : "Compositional Kriging: A Spatial Interpolation Method for Compositional Data". *Mathematical Geology,* no. 33, 2001, pp. 951-966.

[14] J.M. Ver Hoef, N. Cressie: "Multivariable spatial prediction". *Mathematical Geology*, vol. 26, no. 2, 1994, pp. 273-275.

[15] J. Janiczek: "Wieloparametrowa korekcja nieliniowości charakterystyk czujników pomiarowych". *Metrologia wspomagana komputerowo*. MWK, 1999. (in Polish)

[16] J. Janiczek. "Wyznaczanie charakterystyk wieloparametrowych przetworników pomiarowych". *PAK*, vol. 53, nr 9, 2007. (in Polish)

[17] J. Janiczek, M. Zachariasiewicz-Woźniak: "Calibration of a multiparameter sensor: an example of flow meters with unnormalized pneumatic resistance", *Metrol Meas Syst*, vol. X, nr 4, 2003, pp. 411-416.

[18] G. Matheron: "Principles of geostatistics". *Econ. Geol.*, vol. 58, 1963, pp. 1246-1266,

[19] J. Mroczka: "Metrologia w procesie poznania". *Współczesna metrologia – zagadnienia wybrane.* WNT. Warszawa, 2004. (in Polish)

[20] A. Gondek, T. Filipiak: "The effect of double Venturi's structural parameters on the reinforcement value and flow coefficient". *Metrol Meas Syst,* vol. XV, no 3, 2008. pp. 287-300.