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IMPROVED DYNAMIC RANGE FOR MULTI-TONE SIGNAL USING MODEL-BASED PRE-DISTORTION

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Abstract

Some test and measurement applications require higher dynamic range for multi-tone signals than a signal generator can generate. Moreover, for other applications it can be interesting to improve the performance of a "low cost" signal generator. The spectral purity of generated signals can be improved by using pre-distorted base-band signals. In this paper, two model-based pre-distorters are described, where the pre-distorters are based on a static and a dynamic model, respectively. A pre-distorter is used in order to improve the dynamic range. The results, which are based on measurements, show an improved dynamic range for a three-tone signal of approximately 10 dB and an improved ACPR of 5.7 dB for a WCDMA signal.

1. Introduction

In many applications a very accurate test signal is often required, *e.g.* ADC and receiver characterization/testing. Such a signal is often difficult to generate in practice due to the imperfections of the signal generator (SG). By characterizing the SG and using software based methods such as pre-distortion, the dynamic range can be improved. Moreover, in order to reduce production costs it can also be interesting to improve the performance of a "low cost" signal generator to fulfill test requirements. The investigation of the imperfections in the signal generator that is done in this paper is one step to compensate the measurements for these impairments. In Fig. 1, a block scheme of a SG is shown. The IQ-modulator has a number of imperfections. The mixers, digital to analog converters (DACs), and amplifiers are nonlinear, there can be a phase error, θ_0, in the 90 degree phase shifter, the two signal paths may have different gains and there can be DC offsets on one or both of the signal paths.

Some of these imperfections have been studied previously; see for example [1-4]. In addition to the impairments mentioned above the transmitter may exhibit linear distortion that is not discussed here. In [5, 6], a non-parametric, iterative method to

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create pure multi-tone signals using only power measurements was presented. However, the drawbacks of iterative correction are the large number of time-consuming measurements needed and mainly the fact that this method gives a pre-distortion that is unique for a certain signal. In model-based pre-distortion, the pre-distorted signal is computed from a mathematical model of the imperfections mentioned above [7, 8]. It is thereby valid for all kind of signals as long as the signals are within the working conditions for the model.

The paper is organized as follows. In Section 2, an overview of the method and the test setup is given. The theory for gray box modeling is given in Section 3, and the measured results are presented in Section 4. Finally, the paper is concluded in Section 5.

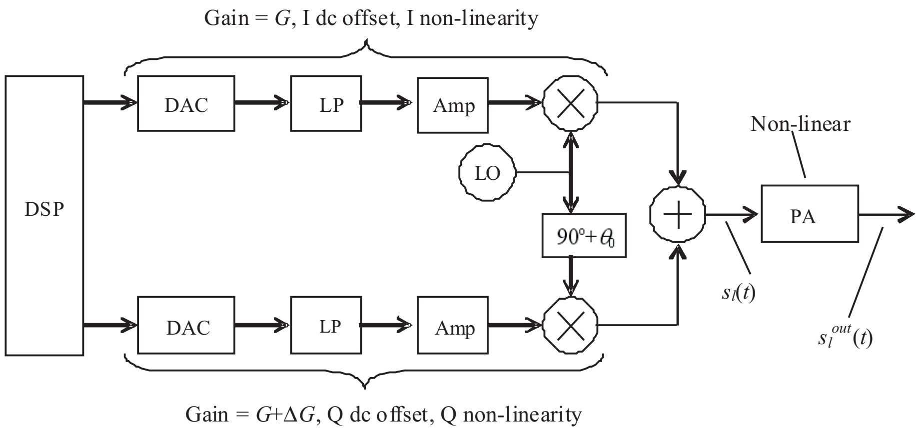

Fig. 1. Imperfections on the signal generation side. There are numerous potential sources of error in a direct up-conversion transmitter, like unequal gain between the two paths, phase error in the phase shifter, nonlinearities and DC offsets.

2. Method

The purpose is to improve the dynamic range of the SG. In order to reduce the distortion due to the intermodulation (IM) products in the SG, a model-based pre-distortion is proposed. To build the pre-distorter, it is necessary to derive a model for the SG. In this paper a grey box modeling approach is used for modeling the SG *i.e.* partial knowledge of its internal architecture, as well as the mathematical analysis given in [4] will be taken in consideration.

The test setup used in this paper work is described in Fig. 2. It consists of a SG (Rohde & Schwarz SMU200A), a signal analyzer (SA) (Rohde & Schwarz FSQ 26), and a personal computer (PC). The SG that will be modeled contains of two independent signal generators in one cabinet. Each channel is equiped with an arbitrary wave generator (AWG) with two digital to analog converters (DAC) up to 100 MHz.

The baseband signals are up-converted directly to the RF in the IQ-modulator. Finally, a power amplifier (PA) increases the signal to the required output power level. More details about the test set up-can be found in [4].

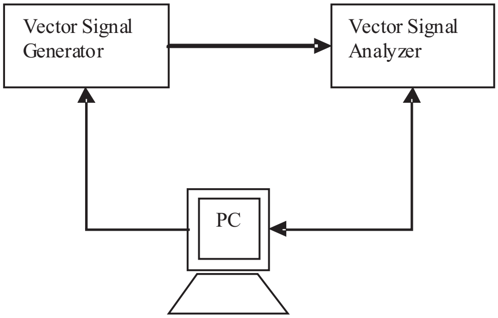

Fig. 2. The measurement system. Test signals are generated by the PC and sent to the signal generator. The output signal is collected by the spectrum analyzer and proceded in the PC.

A single sinusoid is not a suitable signal for excitation when identification of nonlinear terms is required. Instead, if a multi-tone is used, those nonlinear terms can be identified. In this paper, a three-tone technique was used for measuring nonlinear distortion. However, the method used is not limited to a three-tone signal. It can be extended to an arbitrary number of tones.

A model is a representation of a system by using a set of mathematical functions. Usually a system modeling is classified according to how much priori information about the system is available. That is, a white box model is a system where all the information is available, when there is no priori information of the system it is considered as a black box, and if the internal structure of a device is partially known and used in the modeling process it is denominated as a grey box modeling approach.

The number of coefficients in a grey box model can easily grow cumbersomely. In order to reduce the complexity of the model, negligible coefficients have been omitted. This is done by the following method: First the normalized mean square error (NMSE) of the complete model is calculated, thereafter the negligible coefficients are reduced and a new NMSE is computed and compared with the complete model.

Pre-distortion technique relies on the concept of adding signals to the wanted input signal to produce at the output of the overall system the signal of interest without distortion [9]. It is simply to use the inverse of the SG model when generating the signal to the AWG.

3. Theory

Pre-distortion is a technique consisting basically of introducing the inverse, K^{-1}, of the unwanted characteristic of a device under test (DUT), in series with the DUT, to eliminate the distortion introduced by the unwanted DUT's characteristic. In this paper, the DUT is the signal generator. The block diagram of this concept is shown in Fig. 3. If the system is not frequency-dependent, then it can be assumed that the output depends solely on the momentary input signal and this case is noted as the static case.

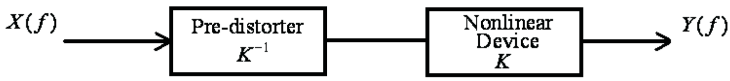

Fig. 3. Pre-distortion main idea.

In the static case, also known as the memory-less case, the applications are mainly of two different types of digital pre-distorters (DPD); look-up table (LUT) based pre-distorter [10] and polynomial pre-distorter [11]. A LUT based pre-distorter stores the pre-distortion coefficients for all input values in the LUT and the incoming signal is multiplied sample by sample with this coefficient. In the polynomial pre-distorter case, the characteristics of the DUT and the pre-distorter are described by polynomial functions. The polynomial coefficients of the pre-distorter are adjusted to compensate the DUT's nonlinearity, resulting in a linear system. In this paper, a polynomial pre-distorter has been used.

A simple approach to model the signal generator is to use a memory-less polynomial model where the output could be estimated using the following equations

$$\hat{y}(n) = \sum\nolimits_{i=1} \theta_i u^i(n). \tag{1}$$

The equation (1) represents the signal generator's system and the task now is to find the values of θ_i coefficients. There is more than one solution for this system, and in order to find the best possible set of parameters θ_i, that estimated the output as close as possible to the real output, a least square estimation method [12] could be used; this method consists of minimizing the squared error given by

$$\frac{\partial \varepsilon}{\partial \theta_i} = 0; \forall i, \tag{2}$$

$$\varepsilon = \sum_{n=0}^{\infty} |e(n)|^2, \tag{3}$$

$$e(n) = y(n) - \hat{y}(n), \tag{4}$$

where $y\ (t\ )$ are $y\ (t\ )$ are the measured and the estimated output, respectively, and $e(n)$ is the estimation error.

A model is said to have memory when the output of the SG is not only a function of the current input, with some constant delay, but it also depends of the previous input values. A nonlinear dynamic system can be described by different model structures, and the choice of structure depends on the application. A commonly used model for power amplifiers (PAs) are the Hammerstein model that consists of a nonlinearity $N(\cdot)$ followed by a linear filter $H(\cdot)$ as shown in the Fig. 4. The output of this model can be written as:

$$y(n) = N(u(n))H(q) = \sum_{m=0}^{M} b_m \sum_{p=1}^{P} h_{2p-1} \left|u(n-m)\right|^{2(p-1)} u(n-m), \tag{5}$$

where:
M is the number of previous samples considered, i.e. the memory length of the model.
P is the polynomial order,
b_i is the coefficient of $N(\cdot)$
h_i is the coefficient of $H(\cdot)$

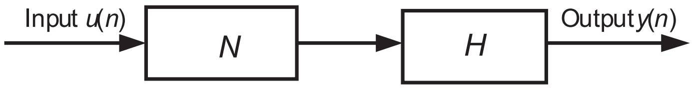

Fig. 4. Block structure of the Hammerstein model.

Since a grey box approach is used to estimate the output of the SG, the information on its internal structure mentioned in Section 1 will be described further. The basic architecture of any AWG can be explained as follows: The samples defining the waveform are stored in a waveform-memory, this stored signal feeds a DAC at the rate defined by the time-resolution of the DAC and finally the signal is filtered and amplified. A block scheme of the SG is illustrated in Fig.1.

The input signal, $s(t)$, which will feed both DACs in the IQ-modulator, can be in general expressed by its envelope $r(t)$ and phase $\varphi(t)$ given by

$$s(t) = \mathrm{Re}\left\{r(t)e^{j(\omega_c+\varphi(t))}\right\}, \tag{6}$$

$$r(t) = \sqrt{x(t)^2 + y(t)^2}, \tag{7}$$

$$\varphi = \arctan\left(\frac{y(t)}{x(y)}\right), \tag{8}$$

where ω_c is the carrier frequency and $x(t)$ and $y(t)$ are the in-phase (I) and quadrature-phase (Q) signals respectively. Now taken in consideration the impairments introduced by the IQ modulator, the complex envelope [13] output signal becomes.

$$s_l(t) = g_I(x(t)) - g_Q(y(t))\sin\theta_0 + jg_Q(y(t))\cos\theta_0, \tag{9}$$

where $g_I(\cdot)$ and $g_Q(\cdot)$ represents the nonlinear transfer functions in I and Q signal paths, respectively, and θ_0 is the phase error in the 90 degree phase shifter. To study these nonlinear functions a good approximation is to express them by polynomial series.

$$\begin{aligned} g_I(x(t)) &= a_0 + a_1 x(t) + a_2 x^2(t) + \ldots \\ g_Q(y(t)) &= b_0 + b_1 y(t) + b_2 y^2(t) + \ldots \end{aligned} \tag{10}$$

Let

$$d(t) = x(t) + jy(t), \tag{11}$$

$$x(t) = d\frac{t(t) + d^*(t)}{2}, \tag{12}$$

$$y(t) = j\frac{d(t) - d^*(t)}{2}, \tag{13}$$

Then, by inserting (10) – (13) into (9) a static model of the IQ modulator will be expressed. The number of coefficients grows rapidly with the order of non-linearity that will be considered. A detailed expression for a fifth-order static IQ modulator model is given in [4] and it contains 13 coefficients. The number of coefficients can be reduced. The method for omitting negligible coefficients is given in Section 2, but there are also other additional methods that can be used. When the method is applied for a SG the phase imbalance and the DC offset in the IQ-modulator can be adjusted in a first step, which simplifies the model and reduces the number of coefficients. An experience from this project is that that it is the preferable method instead of using pre-distortion to compensate for IQ imbalance.

Also the PA which will be present in the SG will contribute to the distortion at the output. A model for the PA which takes AM/AM and AM/PM distortion is given by

$$z(t) = h(r(t))e^{j\varphi(t)}, \tag{14}$$

where $z(t)$ is the PA output and $h(t)$ is the complex function of the combined AM/AM and AM/PM functions represented by g(·) and f(·), respectively.

$$h(t) = g(r(t))e^{f(r(t))}. \tag{15}$$

If the AM/PM distortion is small enough then $g(\cdot)$ and $f(\cdot)$ can be represented as polynomials Taylor expanded and $h(\cdot)$ can be given as polynomial with complex coefficients. The expression can be reduced to include only odd-order coefficient since only inband distortion is considered [4,14,15]. If only the odd-order is considered and the IQ-modulator can be balanced, then (14) analternative can be written as

$$z(t) = h_1 s_l + h_3 |d|^2 d + h_5 |d|^4 d + \ldots \tag{16}$$

However, a static model is usually not sufficient to describe the behavior of the PA. Instead, a dynamic model is required. A suitable model is the Hammerstein model.

Memory effects from the PA can be observed in the in-and output spectrum of the PA, by observing the IM products. For a static system, the amplitude of the IM products should have the same amplitude if the input is a multi-tone with uniform amplitude for all tones, while for a dynamic system there will be an imbalance. This imbalance is observed for the SG, and the asymmetries of upper and lower 3rd order IM-products indicates memory effects [16], thus the memory effects will be considered in the model. Therefore the model will also include a delayed sample in order to include the effects of the LP-filters shown in Fig.1, *i.e.* the output of the SG will be a function of the current and previous input values. The model for the used SG will, after parameter reduction will be

$$s_l^{out}(n) = \sum_{m=0}^{M} \left\{ \begin{array}{l} c_0 + c_{1,m} d(n-m) + c_{2,m} \left(|d(n-m)|^2 d(n-m) + \dfrac{d^{*3}(n-m)}{3} \right) \\ +c_{3,m} \left(|d(n-m)|^2 d^2(n-m) \right) + c_{4,m} |d(n-m)|^2 d^{*2}(n-m) \\ +c_{5,m} |d(n-m)|^4 d(n-m) + c_{6,m} |d(n-m)|^4 d^*(n-m) \end{array} \right\}, \quad (17)$$

where M is the memory depth of the model. To improve the output of the SG a model-based pre-distortion approach will de used. That is, the waveform stored in the waveform memory is pre-processed so that when it passes the IQ-modulator and the PA, it results in the required signal. In this paper, the pre-distorter has the same form as the SG model. The pre-distorter coefficients are calculated so that the distortion and IM products are cancelled out. Equation (18) shows the expression used for the pre-distorter.

$$DPD(n) = \sum_{m=0}^{M} \left\{ \begin{array}{l} q_0 + q_{1,m} d(n-m) + q_{2,m} \left(|d(n-m)|^2 d(n-m) \right) \\ +q_{3,m} \dfrac{d^{*3}(n-m)}{3} + q_{4,m} \left(|d(n-m)|^2 d^2(n-m) | + d(n-m)|^2 d^{*2}(n-m) \right) \\ +q_{5,m} |d(n-m)|^4 d(n-m) + q_{6,m} |d(n-m)|^4 d^*(n-m) \end{array} \right\} \quad (18)$$

4. Results

Initially, two static models with model order five and nine, respectively, will be evaluated. They are evaluated for three sets, or cases, of 3-tones. The cases are: (I) [65, 69, 71] MHz, (II) [67, 70, 71] MHz, and (III) [67, 68, 71] MHz.

In the first part the SG model coefficients were estimated from the first set of frequencies (Case I), and this set of coefficients was used to develop two DPDs of fifth and ninth polynomial degree, respectively. The same digital pre-distorter was applied to three different sets of frequencies (Case I, II, and II), *i.e.* the same pre-distorter was used to correct three different set of frequencies with equal amplitude and equal phase shift.

In the second part, two sets of model coefficients were calculated, one set for each case (Case II and III), that means two pre-distorters were found and each of these pre-distorters was applied to its corresponding set of frequencies, then each case was tried particularly and compared to the results obtained in group A. The amplitude for all frequencies was equal to -5 dBm and a phase shift of 0 ° degree.

The results of all experiments are summarized in Table 1. Moreover, detailed information is presented in frequency plots (Fig. 5) and measurement results (Table 2) for one set of measurements (DPD designed from Case I, applied to Case III): The plots shows the power spectrum of the SG without pre-distortion and with pre-distortions of fifth and ninth degrees.

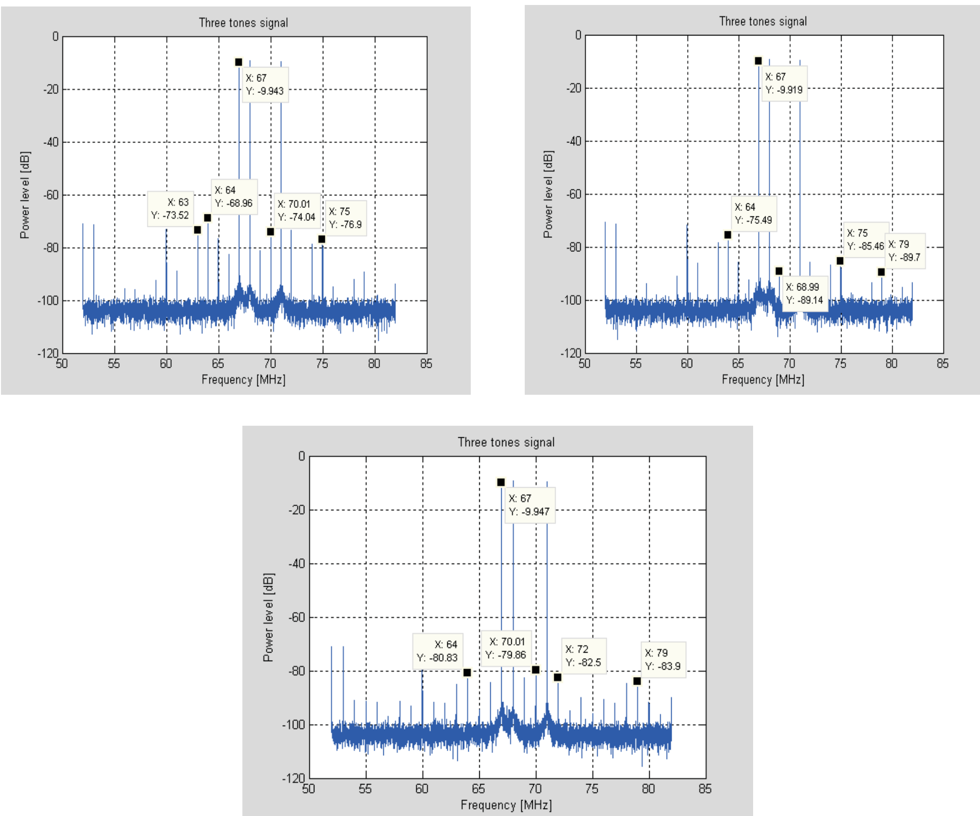

Fig. 5. Power spectrum of the SG; a) before pre-distortion (upper left), b) with a 5th order pre-distorter (upper right), and c) with a 9th order pre-distorter (lower).

Table 1. Summary of the results from all cases.

Case for PD design	Applied to case	Signal level without pre-distortion [dBm]	With pre-distortion of 5^{th} order [dBm]	With pre-distortion of 9^{th} order [dBm]	Reduction in spurious levels (9^{th}) [dB]
I	I	57.90	63.12	69.12	16.68
I	II	58.47	66.14	70.23	16.43
I	III	59.06	65.59	69.96	18.10
II	II	66.15	69.44	68.49	18.23
III	III	58.55	69.00	73.18	17.04

Table 2. Detailed results for the DPD designed from Case I when applied for Case III.

IM frequencies & three tone signal	Frequency [MHz]	Signal level without pre-distortion [dBm]	With pre-distortion of 5^{th} order [dBm]	With pre-distortion of 9^{th} order [dBm]	Reduction in spurious levels (5^{th}) [dB]	Reduction in spurious levels (9^{th}) [dB]
	53	-71.53	-71.22	-71.53	-0.31	0
	60	-73.22	-71.58	-73.74	-1.64	0.52
	60.99	-88.99	-86.21	-91.84	-2.78	2.85
2*f1-f3	63	-73.52	-78.58	-84.98	5.06	11.46
f1+f2-f3 4*f1-3*f2	64	-68.96	-75.49	-80.83	6.53	11.87
2*f2-f3	65	-76.83	-85.88	-94.53	9.05	17.7
2*f1-f2	66	-82.58	-92.44	-84.33	9.86	1.75
	67	-9.943	-9.9	-9.947	-0.043	0.004
	68	-9.358	-9.3	-9.3	-0.058	-0.058
2*f2-f1	68.99	-81.35	-89.14	-82.57	7.79	1.22
f1-f2+f3 3*f2-2*f1	70	-74.04	-97.35	-79.86	23.31	5.82
4*f2-3*f1 4*f2-3*f1	71	-9.6	-9.6	-9.6	0	0
-f1+f2+f3 4*f2-3*f3	72	-73.6	-85.94	-82.5	12.34	8.9
2*f3-f2	74	-78.72	-86.95	-90.12	8.23	11.4
2*f3-f1	75	-76.9	-85.46	-95	8.56	18.1
3*f3-2*f1	79	-89.34	-89.7	-83	0.36	-6.34
Spectral purity [dBc]		59.06	65.59	69.96		

It should be mentioned that the negative values of reduction in spurious values are the consequence of the new additional higher-order distortion from the pre-distorter that wasabsent in the original SG, hence increasing the spurious instead of reducing them.

The results confirmed that the model works fine for a limited set of frequencies and results in an improvement compared to the situation when pre-distortion is not being used. The reason that the model is frequency-dependent are the memory effects present in the system and these effects were not taken into account in these models. In Table 3, one can see that the third-order IM products are not symmetric. That indicates memory effects in the system. Thus, a model with memory should be more proper. For the dynamic DPD a 5^{th} order nonlinearity with one time step memory depth was used. All coefficients in the complex model of the SG given by (16) were estimated. In order to reduce the model complexity the negligible coefficients were removed. The reduced model was evaluated by measuring the NMSE and compared with the NMSE for the full model. The number of parameters was reduced by 50 % within approximately 1 dB preserved for the NMSE.

Table 3. Third order IM products.

IM frequencies & three tone signal	Frequency [MHz]	Signal level without pre-distortion [dBm]
2*f1-f3	63	-73.52
2*f2-f3	65	-76.83
2*f1-f2	66	-82.58
2*f2-f1	69	-81.35
2*f3-f2	74	-78.72
2*f3-f1	75	-76.9

The performances of the memory polynomial pre-distorter were evaluated by measurements on a three tone and a WCDMA signal, respectively. The output power spectra from the SA before and after the pre-distortion are measured. The results from the three-tone measurements are presented in Fig. 6. It shows that the dynamic range has improved by 8.72 dB from 59.98 dBc to 68.70 dBc. That can be compared with the 5^{th} order static model that on average improved the dynamic range by 6.63 dB.

The model was also verified for a WCDMA signal shown in Fig. 7. The improvement obtained in the adjacent channel power ratio (ACPR) is 5.7 dB.

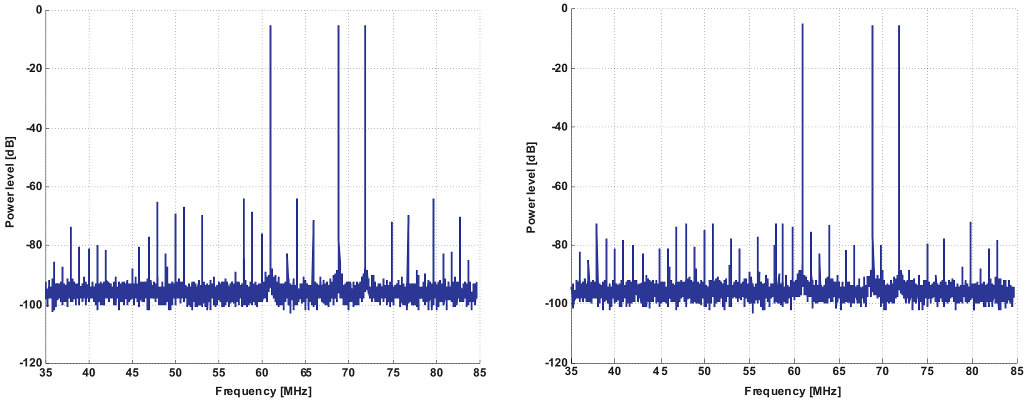

Fig. 6. A three-tone signal with 60 dBc spectral purity before correction (left most) and 69 dBc after (right most).

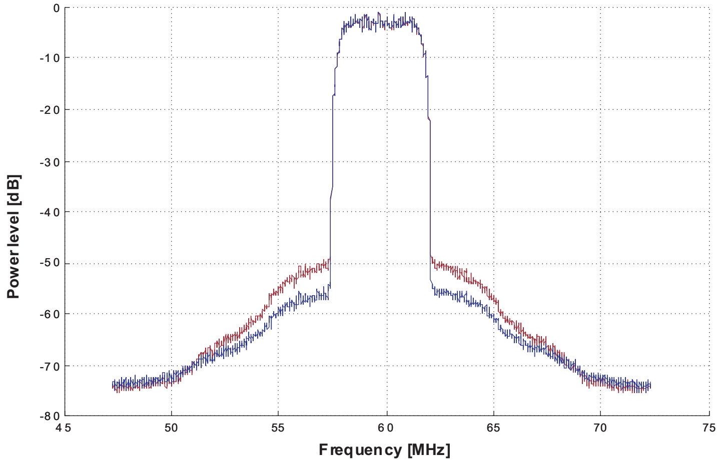

Fig. 7. Improvement in ACPR obtained using pre-distortion of the signal generator. The red curve is the signal before the pre-distortion and blue is the signal after the pre-distortion is applied.

3. Conclusions

In this paper a dynamic grey-box model of a signal generator is presented. The model is then applied to a pre-distorter to eliminate unwanted spectral components and thereby improve the spectral purity when using multi-tone signals. The improvement is a requirement for many test and measurement applications, *e.g.* receiver and ADC characterization and a possibility to reduce cost in other applications. The advantage of model-based pre-distortion is that it is generally applicable for arbitrary signals. Once the model is estimated, it can be used for almost any signal within the working range the model.

The highest value of spectral purity obtained from the static models was approximately 70 dBc for all the cases. A ninth-degree polynomial gave the best results in reduction of the spurious levels with highest value of 18.23 dBm. The results from the dynamic model are based on a fifth-order polynomial model with a memory depth of one time step. The model is reduced to 13 coefficients and applied for pre-distortion for two different signals; one three-tone and one WCDMA signal. The results show an increased dynamic range of 9 dB for the three-tone signal and 5.7 dB lower ACPR for the WCDMA signal.

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