INSULATION RESISTANCE INFLUENCE ON HIGH RESISTANCE HAMON TRANSFER ACCURACY

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Abstract

Accuracy of high resistance Hamon transfers, used for comparison of standard resistors with the ratios of 1:10 and 1:100, depends mainly on insulation leakage. Analysis of the influence of insulation resistance on the transfers accuracies and description of the method of minimizing the insulation leakage current effect by the usage of double insulation are the subject of this paper.

Keywords: high resistance standards, Hamon transfers, insulation resistance, double insulation, analysis.

1. Introduction

The Quantum Hall Resistance (QHR) standard, based on the Quantum Hall Effect (QHE) is currently the most accurate primary standard of resistance [1, 2, 3, 4]. The unit is usually transferred to the standard resistors with nominal value of 100 Ω by a Cryogenic Current Comparator (CCC) [4, 5]. Successive transfer of the unit to standards with higher values, up to 10 kΩ can be performed with the same CCC comparator. For the comparison of resistors with resistance values ranging from 10 kΩ to 1 TΩ, a highly precise bridge, for instance 6000B by Measurement International, can be employed. Above 1 TΩ highly precise instruments for measurements of high resistance values, like 6517A by Keithley or 6500 by Guildline can be used. Uncertainty in transferring the resistance unit depends mainly on the accuracy of the CCC cryogenic comparator, precision of the bridges and instruments for high resistance measurements.

The magnetic CCC with highly sensitive SQUID detector enables comparison of the resistance of primary QHR standard to the standard with nominal value of 100 Ω and the standards of 1 kΩ and 10 kΩ within an accuracy limit of the order of 10^{-8} [7].

The 6000B bridge by Measurements International allows the resistance measurements in the range from 1 kΩ to 1 GΩ with extended uncertainty at a confidence level
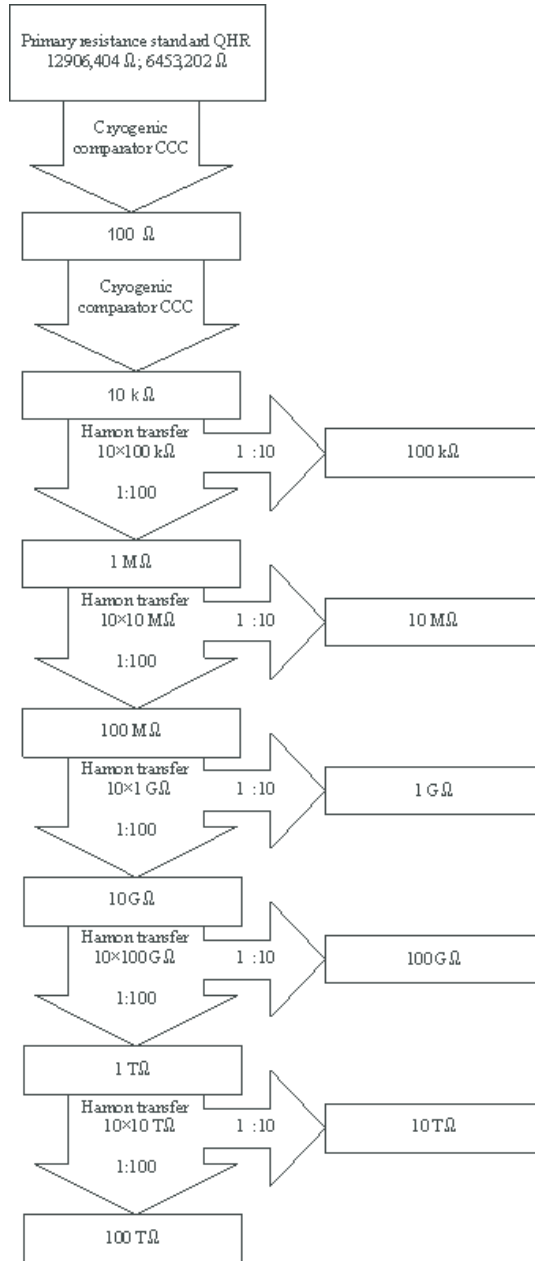
Fig. 1. Scheme of transferring a resistance unit by Hamon transfers toward high values of resistance.

of 0.95, ranging from 0.1 ppm to 5 ppm and, in a special configuration, also in the range of 10 GΩ÷ 1 TΩ with the uncertainty of 20 ÷ 500 ppm [8].

The most accurate measurements of high resistances, ranging from 1 TΩ to 100 TΩ are made with the precise instruments for high resistance measurements like 6517B by Keithley or 6500 by Guildline. The Keithley and Guildline instruments allow the measurement within accuracy limit of 0.4 % ÷ 1.2 % [9] and 0.3 % ÷ 5 % [10], respectively.

To ensure appropriate measurement traceability, the precise bridges and high resistance measuring instruments should be rated with respect to the primary QHR standard. Since the most accurate comparisons of standard values are made by the substitution method when the values of the standards are the same, the comparison of standards with nominal values differing by 10-times and 100-times can be performed with the use of Hamon transfers [1-6]. The ratio of resistance Hamon transfers up to 1M Ω can be estimated with an accuracy of the order of 10^{-7}. For higher values of resistance, the uncertainty increases but is still much lower than the uncertainty of the measurement apparatus used for direct measurements of the standards. The scheme of a system for transferring the resistance unit from the primary resistance standard QHR to high resistance resistors with the use of Hamon transfers is shown in Fig. 1.

For its complete realization in the range from 10 kΩ do 100 TΩ, five Hamon transfers are necessary: 10×100 kΩ, 10×10 MΩ, 10×1 GΩ, 10×100 GΩ, 10×10 TΩ.

2. Hamon transfers

The Hamon transfer consists of 10 precise resistors with the same nominal values, which can be connected in series or in parallel or both (series and parallel) (Fig. 2) [11 - 15]. In the case of a series and parallel arrangement of the transfer resistors, three rows consisting of three resistors connected in series are linked in parallel whereas the tenth resistor is outside the system (Fig. 2c).

The value of i-th transfer resistor

$$R_i = R(1 + d_i), \tag{1}$$

where R is the nominal value of resistors in Hamon transfer, d_i is the relative correction of the value of the i-th resistor in relation to the nominal value. The resistance of ten resistors connected in series:

$$R_S = \sum_{i=1}^{10} R_i = 10R\left(1 + \frac{1}{10}\sum_{i=1}^{10} d_i\right) = 10R\left(1 + \bar{d}\right) \tag{2}$$

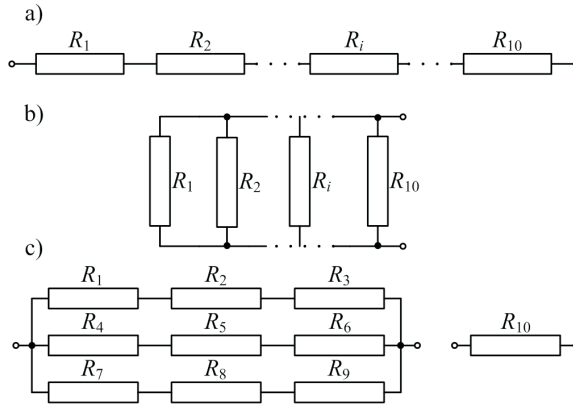and the resistance of a parallel arrangement of the same resistors is

Fig. 2. Hamon transfer in: a) series, b) parallel and c) series-parallel arrangement.

$$R_P = \frac{1}{\sum\limits_{i=1}^{10} \frac{1}{R(1+d_i)}}. \tag{3}$$

Expression (3) after expansion of the denominator into a Maclaurin series and transformation takes the form [11, 12]

$$R_P = \frac{R}{10}\left[1 + \bar{d} - \frac{1}{10}\sum_{i=1}^{10}\left(d_i - \bar{d}\right)^2 + ...\right], \tag{4}$$

where: $\bar{d} = \dfrac{1}{10}\sum\limits_{i=1}^{10} d_i$ is the mean value of relative corrections of the transfer resistors.

In equation (4) elements in power of three and higher were disregarded. Errors of aproximation which result from disregarding these elements depend on the spread of resistance value of resistors used in transfers. For a Hamon transfer, build with 100 kΩ and 0,005% tolerance resistors, the error caused by approximation is of the order of 10^{-15}. Resistors with a higher value of resistance have higher tolerance spread of resistance value, which causes an increase of the approximation error in equation (4). For Hamon transfer built with 10 TΩ and 2% tolerance resistors of approximation error is of the order of 10^{-8}, thus negligible. After dividing the equations (2) and (4) by sides one obtains

$$\frac{R_S}{R_P} = 100\frac{1 + \bar{d}}{1 + \bar{d} - \frac{1}{10}\sum\limits_{i=1}^{10}\left(d_i - \bar{d}\right)^2} \approx 100\left(1 + \frac{1}{10}\sum_{i=1}^{10}\left(d_i - \bar{d}\right)^2\right) = 100\left(1 + d_{S/P}\right). \tag{5}$$

It follows from the above relation that the ratio of series to parallel arrangement of the transfer resistors R_S/R_P amounts to 100 with a minor relative correction $d_{S/P} = \frac{1}{10}\sum_{i=1}^{10}(d_i - \bar{d})^2$. The value of this correction depends on the tolerance of resistors used to build the transfer. For a transfer built with 100 kΩ and 0,005% tolerance resistors this correction is of the order of 10^{-10}, and for 10 TΩ and tolerance 2% tolerance the correction is of the order of 10^{-5}. The resistance of a mixed series-parallel arrangement of the transfer resistors (Fig. 2c)

$$R_{SP} = R\left(1 + \frac{1}{9}\sum_{i=1}^{9}d_i\right) = R\left(1 + \bar{d}'\right) = R\left(1 + \bar{d} - \frac{1}{10}d_{10}\right), \tag{6}$$

where $\bar{d}' = \frac{1}{9}\sum_{i=1}^{9}d_i = \bar{d} - 0{,}1d_{10}$ is the mean value of the relative correction of 9 resistors connected in a mixed arrangement and d_{10} – is the value of the relative correction of the separated 10-th resistor.

Dividing the equations (2) and (6) by sides, one obtains the equation for the ratio of resistance of a series resistors configuration and series-parallel configuration of these resistors

$$\frac{R_S}{R_{SP}} = 10\frac{1 + \bar{d}}{1 + \bar{d} - 0,1d_{10}} \approx 10\left(1 + 0,1d_{10}\right). \tag{7}$$

The approximation used in equation (7) causes an error of the order of 10^{-16}. Whereas the relative correction d_{10} of the separated resistor, for 100 kΩ resistors is of the order of 10^{-6} and for 10 TΩ it is of the order of 10^{-3}. From the above it follows that the ratio of series and series-parallel arrangement of the transfer resistors R_S/R_{SP} is also known and amounts to 10, including the correction of the 10-th rejected resistor. Thus the resistance transfers may enable a very accurate transfer of the values of standard resistances in the ratios 1:10 and 1:100.

3. Insulation leakage

The high resistance resistors in Hamon transfers are connected with insulated terminals or coaxial connectors located on a metal board (ground). The accuracy of the transfers is mainly limited by the insulation leakage to the ground. The scheme of a series connection of the transfer resistors, including the insulation resistance is shown in Fig. 3.
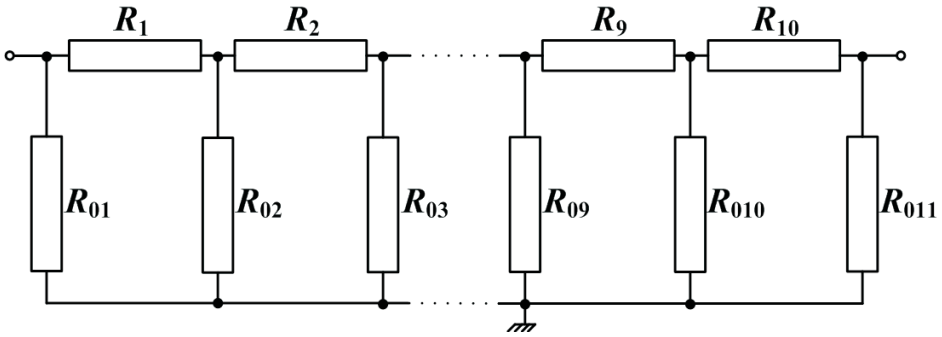
Fig. 3. Hamon transfer in series arrangement including the insulation resistance of the connectors.

The analysis of the effect of insulation resistance on the resultant value of resistance of the transfer in a series arrangement has been presented in the PhD thesis by Skurzak [11] and in the handbook edited by Dudziewicz [12]. In the analyses, the mass and related to it insulation leakage related to it and the fact that the connections with the high resistance transfer were made with screened cables were neglected. The transfer system, including insulation resistances was considered there as a two-terminal network. With such assumptions, the insulation resistance shunted the resistance of the transfer resistors and the resultant resistance of the series arrangement of the transfer resistors was lower than the total resistance of the component resistors. In fact, the resultant transfer resistance in the series arrangement of the resistors, including insulation leakage is higher than the total value of the resistors linked in series . The series arrangement of transfer resistors, including insulation leakage should be considered as a chain of four-terminal networks. To facilitate the analysis, it has been assumed that the resistances of particular resistors R_i in the system are the same and equal to R, whereas the insulation resistances of the connectors are also the same and equal to R_0. With such assumption, the transfer system in Fig. 3 can be represented by the chain of four-terminal networks shown in Fig. 4. To facilitate further considerations, the insulation resistances R_0 of each connector in the system has been replaced by two identical resistances with the value $2R_0$, connected in parallel. There are 10 identical symmetrical four-terminal networks of Π type between the points A and B. Out of the points A and B, there are two insulation leakage resistances with the values $2R_0$ (Fig. 4).
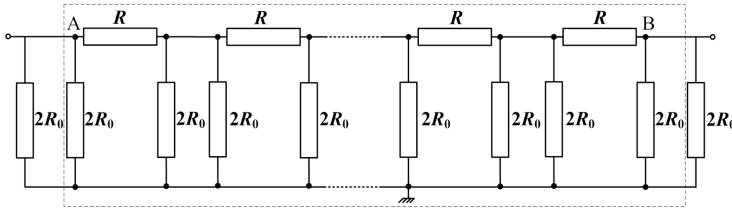
Fig. 4. Transformation of the transfer system from Fig. 3 into a chain of four-terminal networks.

The chain of four-terminal networks can be replaced by one four-terminal network consisting of the components R_{ST} and R_{0C} (Fig. 5). According to four-terminal network theory, the longitudinal resistance of the four-terminal network is

$$R_{ST} = R_C \text{ sh } ng, \tag{8}$$

where

$$R_C = \sqrt{\frac{RR_0}{1 + \frac{R}{4R_0}}}, \tag{9}$$

n is the number of resistors R connected in series, in the case of $n = 10$,

$$g = 2\ln\left(\sqrt{1 + \frac{R}{4R_0}} + \sqrt{\frac{R}{4R_0}}\right). \tag{10}$$

The transverse resistance of the four-terminal network (insulation) in relation to the ground

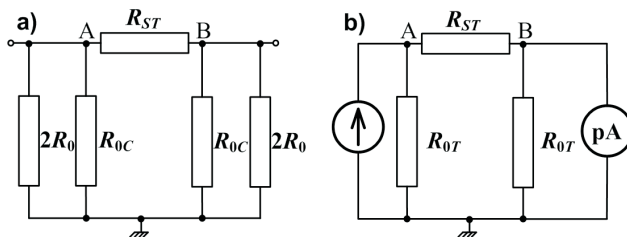$$R_{0C} = \frac{R_C \text{ sh } ng}{\text{ch } ng - 1}. \tag{11}$$

Fig. 5. Replacement of the system from Fig. 4 by one four-terminal network (a) and its representation in a measuring system (b).

The transverse resistances R_{0C} are the resultant of insulation resistances of the four-terminal network. They shunt the 2_{R0} resistances located outside the four-terminal network (Fig. 5a) and can be represented by the resultant of transverse resistances (insulation) R_{0T} (Fig. 5b).

High value resistances can be measured with the instruments for measuring high resistance values, which usually consist of a voltage source and an ammeter, which are linked with a measured resistor in the way shown in Fig. 5b. In the circuit, one resultant resistance of insulation R_{0T} shunts the voltage source, whereas the second one – with the same value, shunts the current measuring instrument (pico-ammeter) with low resistance. If one uses a stable and sufficient enough voltage source then the resistance R_{0T} which shunts the source will have no influence on the measurement. The second resistance R_{0T} which shunts the ammeter is much higher than the internal resistance of the ammeter and can be neglected. Thus both resistances do not influence the measurement result of longitudinal resistance R_{ST}. Also the bridge systems employed in the measurements of high resistances are configured in such a way so that the external resistances of insulation leakages do not influence the result of measurement of resistor resistance value.

The longitudinal resistance R_{ST}, described by the relationship (8) is a resultant resistance of the series arrangement of the transfer resistors including insulation resistance. Its value

$$R_{ST} = R_S(1 - \delta R_S) \tag{12}$$

is burdened with relative error

$$\delta R_S = \frac{10R - R_{ST}}{10R}, \tag{13}$$

with a negative value. From the above it follows that the longitudinal resistance R_{ST} is higher than the total resistance of particular resistors of the transfer R_S.

Linking the resistors in Hamon transfer in parallel, and taking into account their insulation leakage to the ground, one can get the scheme shown in Fig 6a. To facilitate the analysis it was assumed that all transfer resistors R_i have the same resistance values R and that all values of resistance insulation R_{0i} i R'_{0i} are the same and equal to R_0. The system from Fig. 6a can be replaced by the scheme shown in Fig. 6b, where the resultant value of the resistance of transfer resistors connected in parallel is

$$R_{PT} = \frac{1}{10}R \tag{14}$$

and the resultant insulation resistance
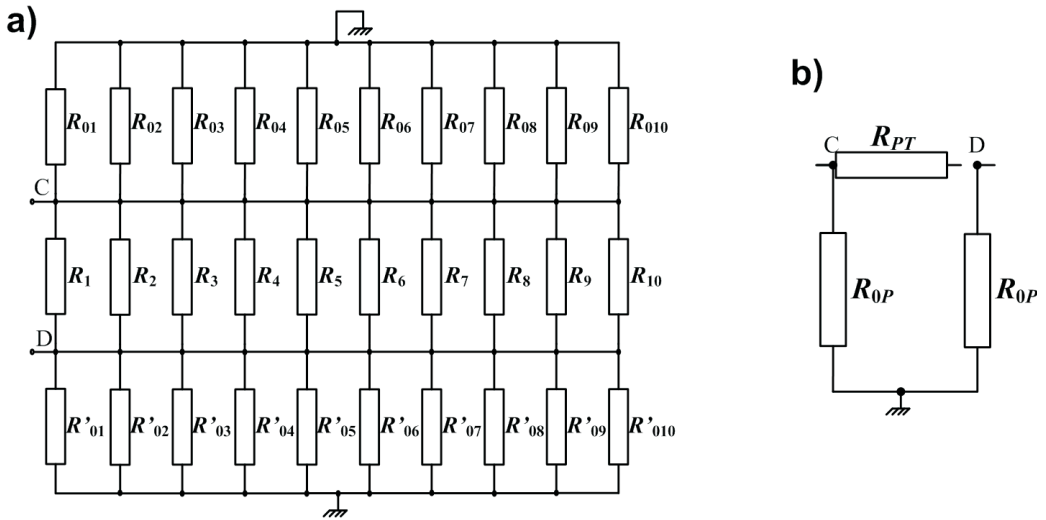
$$R_{0P} = \frac{1}{10}R_0. \tag{15}$$

Fig. 6. Hamon transfer in parallel arrangement including insulation resistance of the connectors (a) and its equivalent circuit (b).

It follows from the above that the resistance of connector insulation does not influence the resultant resistance of the transfer resistors connected in parallel.

The ratio of resistances of high resistance Hamon transfer in series and the parallel arrangements

$$k_{S/P} = \frac{R_S}{R_P}(1 - \delta R_S) \tag{16}$$

is burdened with δ error R_S whose value can be evaluated from the dependence (13). For example, for high resistance transfer 10×10 TΩ and the insulation resistance R_0 of the order of $10^{16}\Omega$, the average error resulting from insulation leakage approaches ca. 1.7%.

In a mixed series-parallel arrangement of the transfer resistors (Fig. 2c), the resultant resistance of a series connection of three resistors R_{3ST} can be estimated from equation (8) for $n=3$. In the parallel arrangement of the three branches, their external leakage resistances do not influence the change in resultant resistance of the series-parallel arrangement. The resultant resistance of the series-parallel arrangement of resistors

$$R_{SPT} = R_{SP}(1 - \delta R_{3S}), \tag{17}$$

where

$$\delta R_{3S} = \frac{3R - R_{3ST}}{3R}. \tag{18}$$

The ratio of resistances of high resistance Hamon transfer in series-parallel arrangement and in parallel arrangement

$$k_{SP/P} = \frac{R_{SP}}{R_P}(1 - \delta R_{3S}) \tag{19}$$

is burdened with error δR_{3S} whose value can be estimated from equation (18).

On the other hand, the ratio of transfer resistances in series and series-parallel arrangements is

$$k_{S/SP} = \frac{R_S}{R_{SP}}(1 - \delta R_S + \delta R_{3S}). \tag{20}$$

From equation (20) it can be inferred that the errors following from $k_{S/SP}$ transfer ratio caused by leakages of insulation resistances in series and series-parallel, partly compensate each other.
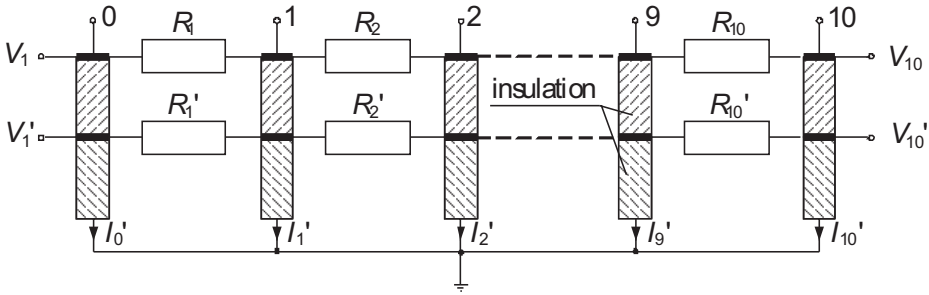
4. Double insulation

Fig. 7. High resistance Hamon transfer with double insulation: $R_1 \div R_{10}$ – resistors of the main transfer, $R'_1 \div R'_{10}$ – resistors of auxiliary transfer, $I_0' \div I_{10}'$ – leakage currents, $V_1 \div V_{10}$ – potentials on the terminals of the main transfer, $V_1' \div V_{10}'$ – potentials of the shields of the connections with double insulation of the auxiliary transfer.

To reduce the effect of leakage of insulation resistance coaxial connectors with double insulation should be used. The scheme of series connection of the transfer resistors with the connectors with double insulation is shown in Fig. 7. The potentials V_1 (terminal 0) and V_1' should be of the same value, usually equal to the source of measuring voltage (Fig. 8a). Terminal 10, with the potential V_{10} is connected in series with the current measuring instrument and the shield with potential V_{10}' is linked

directly to the ground or through a resistor with the resistance equal to the resistance of the measuring instrument.

The influence of the double insulation on leakage currents is shown in Fig. 8a. The leakage current

$$I_0 = \frac{V_A - V_B}{R_0}. \tag{21}$$

Current $I_0 \rightarrow 0$ if $V_A \rightarrow V_B$ or $R_0 \rightarrow \infty$. For finite value of insulation resistance R_0, the potential of connection shield V_B should be increased to the value of potential V_A. It is possible to fulfill if this requirement if the values of the resistors satisfy the condition

$$\frac{R_1}{R'_1} = \frac{R_2}{R'_2}, \tag{22}$$

the resistors R_1 and R_1' are linked to the same voltage source and the currents $I_1 = I_2$ and $I_1' = I_2'$. Due to some spreading in resistance values of the resistors, the condition (22) can be fulfilled to some extent only. The system with double insulation will perform like the system with single insulation (Fig. 8b) with insulation resistance value

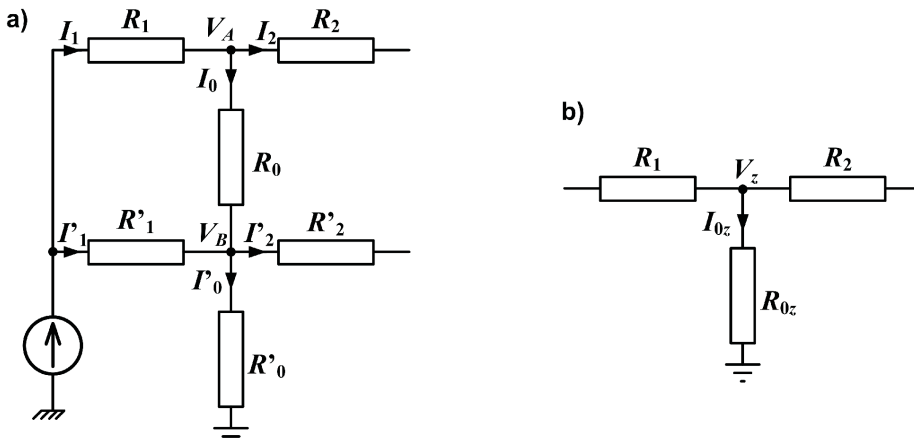$$R_{0z} = R_0 \frac{V_A}{V_A - V_B}. \tag{23}$$

Fig. 8. Arrangement of resistors with increased shield potential (a) and equivalent circuit (b).

The double insulation, in the case of application of resistors with 1 % spreading in resistance, enables at least 100 times reduction of leakage currents. It results also in a ca. 100-fold times reduction of the transfer resistance error.

5. Conclusions

The accuracy of high resistance Hamon transfers is mainly affected by the insulation leakage. This effect can be reduced if in the transfer system the connectors with the best insulation will be used and the resistors will be connected directly to the connectors. The resistors cannot be placed on any insulating boards, for example on printed circuits boards. The high resistance Hamon transfers are connected with shielded cables with very high insulation resistance. If one analyzes the influence of the insulation resistance, one should consider the transfer as a four-terminal network and not a two-terminal network.

For the transfers with the highest resistance values, double insulation should be used (Fig. 7) with increased shield potential and double-shielded connectors and leads.

The work of the authors on the construction of the high resistance Hamon transfers, which are indispensable for resistance unit transfer from the QHR primary standard, are currently in the initial phase. For proper design it is required to analyze all factors which can influence the accuracy of the Hamon transfers. Research on similar resistance transfers is also carried out in The National Institute of Standards and Technology (NIST) in USA [13-15], with which authors co-operate.

Acknowledgements

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