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SEVERAL APPROACHES TO ADC TRANSFER FUNCTION APPROXIMATION AND THEIR APPLICATION FOR ADC NON-LINEARITY CORRECTION

The performance of current electronic devices is mostly limited by analog front-end and analog-to-digital converter's (ADC) actual parameters. One of the most important parameters is ADC nonlinearity. The correction of this imperfection can be accomplished in the output data but only if the nonlinearity is well characterized. Many approaches to ADC characterization have been proposed in scientific articles in the last several years. In this paper three different approximations of ADC low-frequency non-linearity (common polynomials, Chebyshev polynomials and Fourier series) were analyzed and the practical applicability, approximation accuracy and noise sensitivity were investigated. The first results of nonlinearity correction were presented, too.

Keywords: analog-to-digital converter, ADC nonlinearity, transfer function approximation, nonlinearity correction.

1. INTRODUCTION

One of the most important parameters of analog-to-digital converters is the nonlinearity. ADC nonlinearity is inherently described by the Integral Non-Linearity *INL*(n), so the difference of ADC ideal and actual transfer function, where n is the input code. However, only a single number for the *INL*(n) is often presented in manufacturer's datasheets (*INL*) that stands for the maximum value of the *INL*(n) curve.

ADC nonlinearity particularly depends on input signal parameters. Even though the nonlinearity shows strong dependency on signal frequency, this effect can be mostly neglected for signal frequencies significantly lower than ADC sampling frequency. Consequently, ADC nonlinearity can be described only as a function of signal input level.

The nonlinearity causes harmonic distortion in the digitized signal, which can be expressed in the frequency domain by the *THD* (Total Harmonic Distortion) parameter. This is also a single value parameter, but the frequency spectrum can provide similar

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information as the curve of the $INL(n)$ in the code domain (amplitudes and phases of harmonic components). However, the $INL(n)$ is more advantageous in the case when the LUT-based correction of an ADC transfer function is demanded.

The curve of the $INL(n)$ can be split into the low-code frequency component (LCF) and the high-frequency component (HCF). The LCF dominates in practice hence the HCF is not needed for rough approximation of the $INL(n)$. Unfortunately an exact break-point between those two components does not exist [1], [2], [3], [4].

Histogram test described in IEEE 1241 Standard [5] enables the computation of both LCF and HCF parts of the $INL(n)$; but, this method demands a huge number of samples in a record to achieve reasonable confidence levels. When only the LCF is demanded, significantly lower number of samples is sufficient for an accurate estimation of the $INL(n)$. Common frequency spectrum of the acquired signal can be used for the computation of this curve then.

The methods of ADC non-linearity approximation investigated in this paper calculate the coefficients from the frequency domain [6], [7]. Three types of approximations (common polynomials [1], Chebyshev polynomials [8] and Fourier series [9]) were described in section 2. The evaluation of the proposed approximations was performed in section 3 (applicability, accuracy and noise sensitivity) and non-linearity correction was tested in section 4.

2. APPROXIMATION OF INL CURVE

2.1. Common polynomials

In the case of common polynomials [1] the $INL(n)$ is approximated by

$$INL(n) = \sum_{k=0}^{K_{\max}} a_k x^k(n). \tag{1}$$

where a_k are the coefficients of the nonlinearity up to the maximum order, K_{max}, which is the highest harmonic component considered. Index k in summation starts from zero because all even order nonlinearity coefficients induce a DC level as well as all odd nonlinearity coefficients induce a contribution to the first order component.

The relation between nonlinearity coefficients, a_k, and the amplitude of h^{th} harmonic components, Y_h, in the frequency spectrum is done by the formula [10]

$$Y_h = \sum_{n=0}^{s} \frac{(2n+h)!}{2^{2n+h-1} n!(n+h)!} a_{2n+h} X_1^{2n+h}, \tag{2}$$

where X_1 is the amplitude of the input fundamental component, $s = (P–h)/2$ for $P–h$ even and $s = (P–h–1)/2$ for $P–h$ odd, and P is the highest computed power. This relation can be expressed in matrix form as

$$\mathbf{Y} = \mathbf{X} \cdot \mathbf{a}. \tag{3}$$

E.g. this equation for the 3^{rd} order approximation is [1]

$$\begin{bmatrix} Y_0 \\ Y_1 \\ Y_2 \\ Y_3 \end{bmatrix} = \begin{bmatrix} 2 & 0 & X_1^2 & 0 \\ 0 & X_1 & 0 & \frac{3}{4}X_1^3 \\ 0 & 0 & \frac{1}{2}X_1^2 & 0 \\ 0 & 0 & 0 & \frac{1}{4}X_1^3 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix}. \tag{4}$$

Polynomial coefficients can be determined from (3) by inverting matrix $\mathbf{X}$

$$\mathbf{a} = \mathbf{X}^{-1}\mathbf{Y}. \tag{5}$$

2.2. Chebyshev polynomials

Assuming terminal based $INL(n)$ and given Chebyshev polynomials [8] of the first kind $T_h(\cos(x)) = \cos(hx)$, the $INL(n)$ can be approximated by

$$INL(n) = \frac{c_0}{2} + \sum_{h=2}^{\mathrm{H}_{max}} c_h T_h(n). \tag{6}$$

where c_h are the coefficients of the nonlinearity up to the maximum order H_{max}, (same as above). The summation coefficient starts from two due to the orthogonality on the interval [1; 1] (e.i. one polynomial influences only one spectral component). This is the basic and important characteristic of Chebyshev polynomials. The relation of coefficients and harmonic components can be expressed similarly to (3) as

$$\mathbf{Y} = \mathbf{T} \cdot \mathbf{c}. \tag{7}$$

but unlike to (4) in the example of 3^{rd} order approximation $\mathbf{T}$ is only a diagonal matrix [11]

$$\begin{bmatrix} Y_0 \\ Y_1 \\ Y_2 \\ Y_3 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & T_1 & 0 & 0 \\ 0 & 0 & T_2 & 0 \\ 0 & 0 & 0 & T_3 \end{bmatrix} \begin{bmatrix} c_0 \\ c_1 \\ c_2 \\ c_3 \end{bmatrix}. \tag{8}$$

The determination of the matrix of coefficients $\mathbf{c}$ is obvious.

2.3. Fourier series

In the case of Fourier series [9] the *INL*(n) is approximated by

$$INL(n) = \frac{a_0}{2} + \sum_{k=0}^{2^N-1} \left[a_k \cos(\frac{2\pi}{2^N} nk) + b_k \sin(\frac{2\pi}{2^N} nk) \right]. \tag{9}$$

where a_k and b_k of a known *INL*(n) can be found using the well-known expressions

$$\begin{aligned} a_k &= \frac{1}{2^B} \sum_{n=0}^{2^B-1} INL(n) \cos(\frac{2\pi}{B} nk),\ k \in \{0, 1, ..., 2^B - 1\} \\ b_k &= \frac{1}{2^B} \sum_{n=0}^{2^B-1} INL(n) \sin(\frac{2\pi}{B} nk),\ k \in \{1, ..., 2^B - 1\} \end{aligned} . \tag{10}$$

where 2^B–1 is the number of transition levels of a B bit ADC and k is the index of the coefficients. The *INL*(n), from which a_k and b_k are calculated, is considered to be periodical; from this reason it is better to use the terminal-based *INL*(n) [5] in order to minimize the step between the start and end of the *INL*(n).

For the input normalized signal

$$x(m) = \frac{2^B}{X_{FS}} \left(X_1 \cos(\theta_m) + X_0 \right). \tag{11}$$

the output signal is

$$y(m) = x(m) + \frac{a_0}{2} + \sum_{k=1}^{2^B-1} a_k \cos\left[\frac{2\pi k}{X_{FS}} (X_1 \cos\theta_m + X_0) \right] + \sum_{k=1}^{2^B-1} b_k \sin\left[\frac{2\pi k}{X_{FS}} (X_1 \cos\theta_m + X_0) \right] \tag{12}$$

that can be expressed by

$$y(m) = x(m) + \sum_{h=0}^{\mathrm{H}_{max}} Y_h \cos(h\theta_m). \tag{13}$$

where Y_h represents the h^{th} output harmonic component that can be expressed by means of Bessel functions of the first kind $J_h(.)$ with order h.

$$\begin{aligned} Y_{2h} &= 2(-1)^h \sum_{k=1}^{2^B-1} \left(a_k \cos\frac{2\pi k X_0}{X_{FS}} + b_k \sin\frac{2\pi k X_0}{X_{FS}} \right) J_{2h}\left(\frac{2\pi k X_1}{X_{FS}} \right) \quad h \geq 1 \\ Y_{2h+1} &= 2(-1)^h \sum_{k=1}^{2^B-1} \left(b_k \cos\frac{2\pi k X_0}{X_{FS}} - a_k \sin\frac{2\pi k X_0}{X_{FS}} \right) J_{2h+1}\left(\frac{2\pi k X_1}{X_{FS}} \right) \quad h \geq 0 \end{aligned} . \tag{14}$$

X_{FS} is ADC full-scale range, X_0 is input signal DC offset and θ_m is input signal sampling phase. The coefficients a_k, b_k can be determined from equations (14) expressed in matrix form.

3. EVALUATION OF THE PROPOSED APPROXIMATIONS

3.1. Applicability

Practical applicability of the approximations mentioned above was tested on a real curve $INL^{real}(n)$, which was obtained by measuring 14-bit digitizer (National Instruments PXI 5122). The real $INL^{real}(n)$ spaned around 80% of whole full scale range of the tested ADC. A simulated cosine wave $x(m)$ was input to this nonlinearity. Then, the output signal is done by

$$y(m) = x(m) + INL^{real}(x(m)). \tag{15}$$

The coefficients of each approximation were calculated from the frequency spectrum of output signal $y(m)$ and used for the reconstruction of the $INL^{approx}(n)$. The results are shown in Fig. 1. Simulations showed that common polynomials can approximate only a rough plot of the $INL(n)$ curve up to approximately 150 coefficients (Fig. 1a, 1b). The quality of approximations by Chebyshev polynomials and Fourier series is comparable. Fourier series can approximate the $INL(n)$ curve in more detail (see Fig. 1c).

The advantage of Fourier series, unlike the other two polynomial based approximations, is the capability to approximate sharp transitions in the $INL(n)$ curve. On the other hand, Fourier coefficients lead to very complex but stable solutions. However in most practical applications, the number of coefficients up to 10 is sufficient and all mentioned approximations can be applied.

3.2. Accuracy evaluation

The approximated $INL^{approx}(n)$ calculated from the frequency spectrum of $y(m)$ were compared with the actual $INL^{real}(n)$ measured by histogram method. The accuracy of approximations was evaluated by means of the Mean Square Error MSE (16) and the absolute value of maximum error E_{max} (17) defined as

$$MSE = \frac{1}{N}\sum_{n=0}^{N-1}\left(INL^{real}(n) - INL^{approx}(n)\right)^2. \tag{16}$$

$$\mathrm{E}_{max} = \max\left|INL^{real}(n) - INL^{approx}(n)\right|. \tag{17}$$

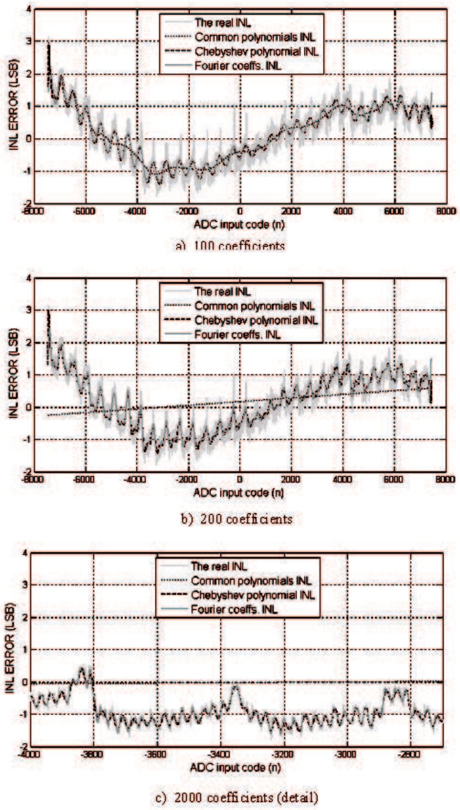

Fig. 1. $INL(n)$ approximations.

where N is the length of both $INL(n)$ curves chosen as $N = 2^B–1$, B is the nominal number of bits of the tested ADC. The results with respect to the number of coefficients are shown in Table 1.

Table 1. Mean square error (MSE) and maximum error (E_{max}) for different numbers of coefficients, no noise added.

	100 coefficients		200 coefficients		2000 coefficients	
	MSE (LSB^2)	E_{max} (LSB)	MSE (LSB^2)	E_{max} (LSB)	MSE (LSB^2)	E_{max} (LSB)
Common polynomials	0.08	1.53	0.65	3.39	0.65	3.39
Chebyshev polynomials	0.05	1.39	0.04	1.20	0.01	0.50
Fourier series	0.05	1.56 (1.32)*	0.04	1.49 (1.01)*	0.01	0.50

For small number of coefficients – in this case 100 coefficients (see Fig. 2a) – all approximations successfully fitted the nonlinearity and the values of MSE and E_{max} achieved comparable levels. When the number of estimated coefficients reached roughly

170, the approximation by common polynomials failed and it provided only a straight line as the result (see Fig. 1b). Consequently the value of MSE and E_{max} parameters was much higher in relation with Chebyschev polynomials and Fourier series when considering 200 coefficients (see Fig. 2b). For the number of 2000 coefficients (see Fig. 2c) the MSE and E_{max} were still lower and at comparable values for Chebyshev polynomials and Fourier series.

It is important to mention that in the case of Fourier series the most error-prone part of the approximated $INL(n)$ is the beginning and the end of the $INL^{approx}(n)$ curve. This is caused by non-strictly continuous periodical extension in higher derivations of the $INL^{approx}(n)$ curve. If the beginning and the end of each approximation are omitted from error calculation, the MSE and E_{max} are even smaller (the values denoted by * in Table 1) for 100 and 200 coefficients. The number of 2000 coefficients is sufficient for good approximation of higher derivations in the $INL^{approx}(n)$ curve.

In general, the performance of Chebyshev polynomials and Fourier series is comparable, although the complexity of Chebyshev polynomials is smaller and they are consequently easier to implement.

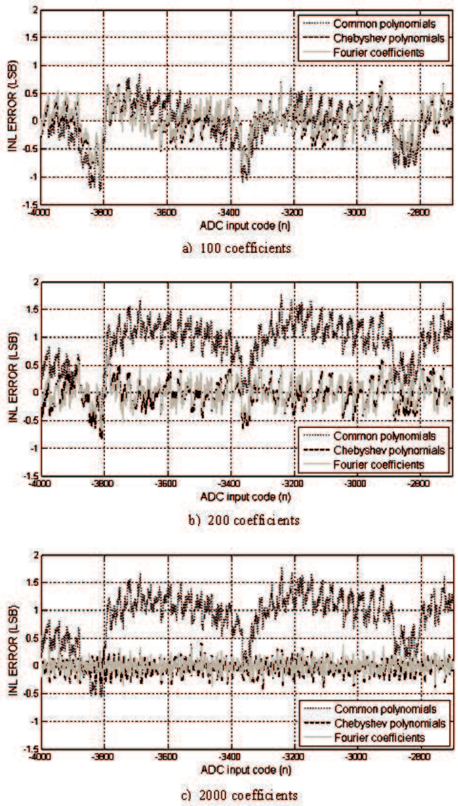

Fig. 2. Errors of $INL(n)$ approximations (detail).

3.3. Noise sensitivity

One of the most important parameters of every algorithm is the sensitivity to noise. The analysis of the influence of noise to the proposed approximations was performed on model structure shown in Fig. 3, where $x(m)$ is the input signal, $y(m)$ is the output signal, which is the output code of an ADC, and $e(m)$ is the noise added to the converted signal. White noise with normal distribution with variance σ^2 was considered.

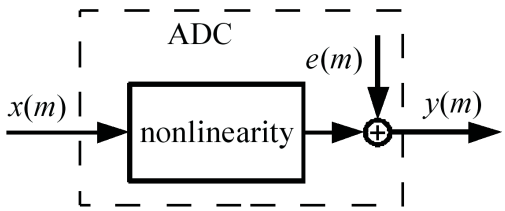

Fig. 3. ADC model used in the analysis.

The procedure of noise sensitivity evaluation (simulated in Matlab) consisted of in the computation of approximation coefficients from the complex frequency spectrum of the output signal $y(m)$ with additive noise. The differences of the $INL^{approx}(n)$ reconstructed from the calculated coefficients and the actual $INL^{real}(n)$ are presented in Fig. 4.

The results of all approximations for 100 coefficients are roughly comparable; only the sensitivity of mean square error to noise of the approximation by common polynomials was low (see Fig. 4a). Since this approximation failed at higher number of coefficients it was not plotted in further figures. The sensitivity to noise of the approximation applying Chebyshev polynomials and Fourier series was comparable for higher number of coefficients (see Fig. 4b, 4c). However, the approximation by Fourier series gave the lowest maximum errors for significant noise. Generally the higher number of coefficients is used the more are the approximations sensitive to additive noise.

4. NONLINEARITY CORRECTION BASED ON $INL(N)$ APPROXIMATIONS

When the coefficients and consequently the $INL(n)$ curve are known, a simple look-up-table (LUT) correction for the nonlinearity can be performed. For this purpose a transfer function $TF(n)$ has to be calculated as the sum of a straight line n and the $INL^{approx}(n)$. If no difference between the $INL^{approx}(n)$ and $INL^{approx}(n+1)$ is bigger than 0.5 LSB for any n, the transfer function $z = TF(n)$ is purely monotone and its inverse function $n = \mathrm{TF}^{-1}(z)$ does exist.

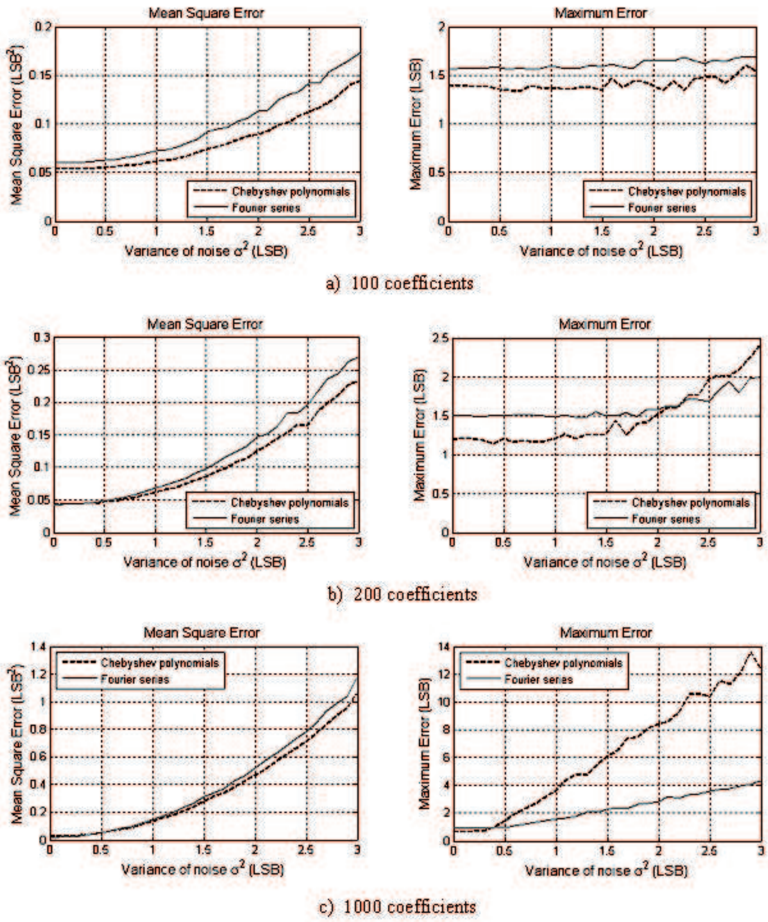

Fig. 4. Errors of $INL(n)$ approximations in dependence on additive noise.

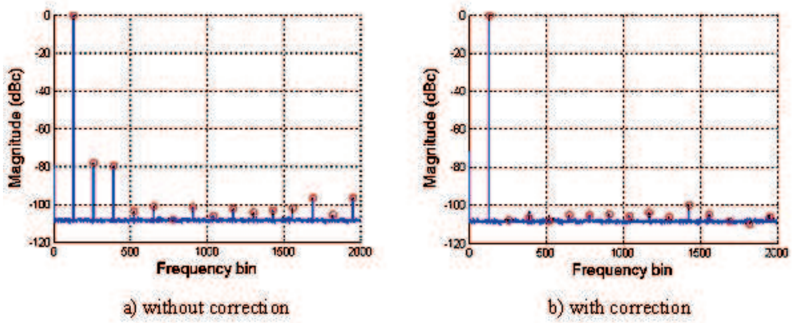

Fig. 5. Frequency spectra of output signal.

The first experimental ADC corrections were performed on the real curve $INL^{real}(n)$ of the NI PXI 5122 digitizer (see above). Chebyshev approximation of the $INL(n)$ by

100 coefficients was computed for a simulated input cosine wave with additive noise of 2 LSB standard deviation. The LUT was computed numerically: noninteger n was searched that corresponded to integer z. Frequency spectra of the output signal without and with this correction are shown in Fig. 5.

5. FURTHER WORK

All analyses carried out in this paper employ coherent sampling that is in practice difficult to achieve. In case of non-coherent sampling leakage occurs and windowing is unavoidable. The influence of non-coherency and window types should be evaluated.

The correction of ADC nonlinearity performed so far was based on a simple LUT computed numerically. In the next step the inverse transfer function will be expressed analytically and applied for on-line correction.

6. CONCLUSIONS

Accuracy and noise sensitivity of three types of approximations (common polynomials, Chebyshev polynomials and Fourier series) of low-frequency ADC nonlinearity were analyzed. Coefficients of all approximations were computed from frequency spectra of simulated signals applied on the nonlinearity of a real 14 bit ADC. A simple LUT based correction of ADC nonlinearity was performed.

Accuracy evaluation showed that all approximations perform comparably for small orders (number of coefficients). For higher number of coefficients the approximation by common polynomials fails while Chebyshev polynomials and Fourier series perform similarly and well. Fourier series are capable to follow sharp transitions in ADC transfer curve and reach the lowest maximal errors. The disadvantage of this approximation is its complexity and the necessity of periodical extension of the nonlinearity. The higher number of coefficients is used for all approximations the more are the approximations sensitive to additive noise. The correction of ADC nonlinearity enabled a significant suppression of ADC harmonic distortion in the simulated signal.

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