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UTILIZATION OF LEVENBERG-MARQUARDT'S METHOD FOR IDENTIFICATION OF THE ELECTRONIC CURRENT TRANSDUCER WITH A HALL EFFECT SENSOR IN A FEEDBACK LOOP

The utilization of Levenberg-Marquardt's method (LM) for the identification of an unknown model parameters of the electronic current transducer with a Hall effect sensor in the negative feedback loop has been described. The saturation reasons of the electronic block in the transducer model have been also taken into consideration which made it possible to use the model in error analysis of the electronic transducer while deformed signals are being measured in the power electronics and automation.

Key words: electronic current transducer, identification methods, Levenberg-Marquardt's method

1. INTRODUCTION

Nowadays electronic current transducers with magnetic flux compensation are very often applied in power electronic industrial devices [13]. Reasons for the increased popularity of electronic current transducers are: their simplicity, construction tested in practice and low price for relatively high measurement accuracy (the typical error is below 1% for the measurement of direct currents and currents in the frequency range of a few kHz). Moreover, fast development of power electronic devices induces continuous demand for measurement devices with better and better metrological properties.

Electronic current transducers have also disadvantages such as: the occurrence of a small unbalanced current of the electronic circuit at zero input current, the necessity of the external supply source stabilized within ±5%, significant accuracy deterioration beyond the acceptable operating temperature range (-40÷85°C) and the occurrence of serious errors beyond the electronic block frequency bandwidth [7, 8, 9].

Taking into account numerous applications of electronic current transducers in the power electronics [13, 14] and the fact that previous electronic transducer models [5, 10, 13] have appeared to be insufficient during simulation tests for distorted primary currents whose individual harmonics lay within the passband of the electronic block, the authors have constructed its new model [11]. This new model enables to analyze metrological properties of electronic current transducers while all harmonics of the primary current lie within the operating passband of the electronic block.

Some parameters of a new electronic current transducer model are presented in the specification of these electronic transducers, whereas the remaining parameters are defined *a priori*. To eliminate this inconvenience, the authors have suggested an identification method for these unknown parameters. To realize the identification process, the user will have to perform a series of measurements when a sinusoidal current is forced in the primary circuit. In each measurement the frequency of the primary current ought to belong to the bandwidth of the electronic block and its amplitude should not exceed the value of rated current given in the specification of the electronic transducer.

2. MODEL OF AN ELECTRONIC CURRENT TRANSDUCER WITH A HALL EFFECT SENSOR IN THE NEGATIVE FEEDBACK LOOP

Only the most important information concerning the new mathematical model of the electronic current transducer will be presented here. More information can be found in works: [5, 9, 11, 13, 14].

Electronic current transducers are adapted to transform direct currents, alternating currents, distorted currents and impulse currents with galvanic separation between the primary circuit (power circuits) and the secondary circuit (measurement devices). Figure 1 presents a functional diagram of an electronic current transducer.

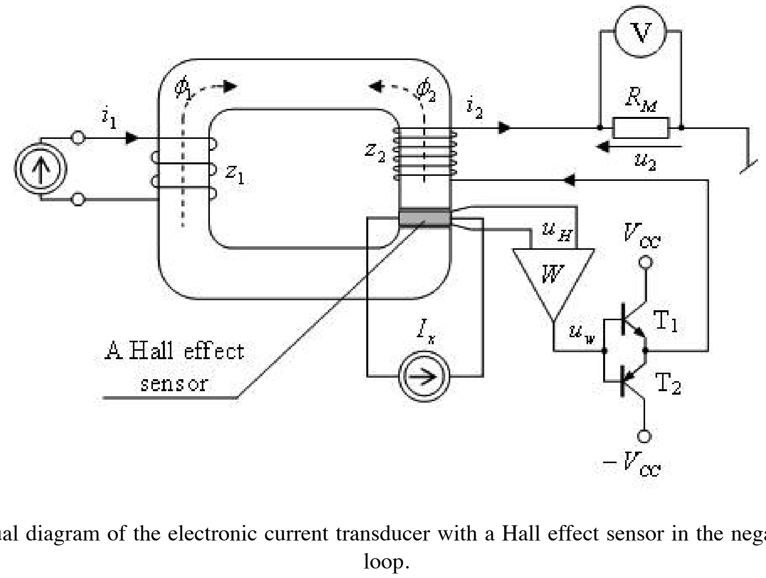

Fig. 1. A visual diagram of the electronic current transducer with a Hall effect sensor in the negative feedback loop.

The principle of operation of electronic current transducers with a Hall effect sensor in the negative feedback loop consists in the measurement of the magnetic flux that is forced in the primary circuit (Fig. 1). The measurement of the primary current is realized without loading the primary circuit and power loss in this circuit.

The worked out mathematical model of an electronic current transducer is presented in Fig. 2.

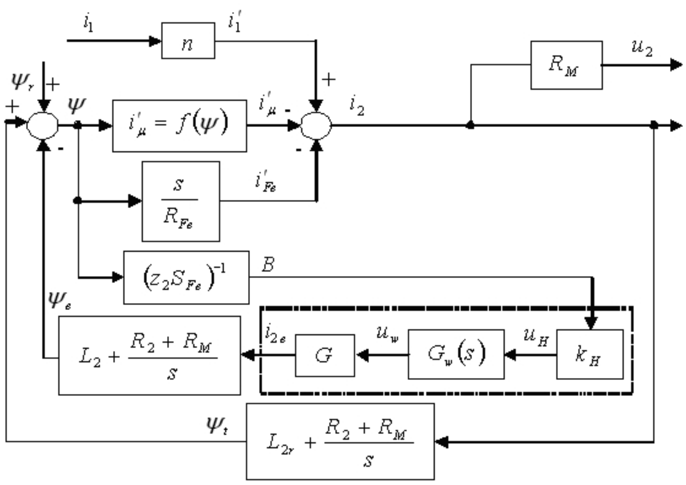

Fig. 2. A block diagram of the electronic current transducer model [11].

The model in Fig. 2 is a combination of the current transformer model [4] with the electronic circuit model of the electronic transducer (the rectangle drawn with the dashed line in Fig. 2).

The block diagram (Fig. 2) includes a block $i_\mu' = f(\psi)$ which represents magnetic core properties. The block is determined by the following equation:

$$i_\mu' = g\left(\frac{\psi}{S_{Fe} z_2}\right)\frac{l_{Fe}}{z_2} + \frac{\psi}{S_{Fe} z_2^2 \mu_0} l_0 = f(\psi) \;, \tag{1}$$

where: i'_μ is the magnetizing current, ψ is the magnetic flux, $B = \psi / S_{Fe} z_2$ is the magnetic induction, $g(B) = H$ is the nonhysteric B-H curve, S_{Fe} is the core cross section, l_{Fe}, l_0 are respectively average lengths of magnetic circuit and air-gap, z_2 is the number of turns in secondary circuit, μ_0 is the permeability of vacuum.

Other parameters: R_1, R_2, L_2 - representing respectively resistances and inductances of primary and secondary circuits, R_{Fe}, R_M, L_{2r} - determining respectively magnetic core losses, the value of the measurement resistor, the inductance associated with the dispersion flux ϕ_{2r}, the parameter $\psi_r = B_r z_2 S_0$ combined with the residual induction B_r and $n = z_1/z_2$ representing the transformer turns ratio. The electronic block was modeled by the following blocks: G, k_H, $G_w(s)$, that adequately represent the transconductance of transistors T_1 or T_2, the constant of the Hall effect sensor and the first order transfer function of the operational amplifier defined as:

$$G_w(s) = \frac{A_{v0}}{1 + \frac{s}{\omega_g}} \quad , \tag{2}$$

where: A_{v0} - is the DC amplification coefficient, ω_g is the 3dB-cutoff angular frequency of the amplitude characteristic of the amplifier.

The Hall effect sensor is placed in the air-gap of the magnetic core and it produces a Hall voltage - u_H that is proportional to the magnetic induction B that appears in the magnetic core (Fig. 1). The Hall voltage feeds the operational amplifier that controls the output voltage – u_w. Afterwards the output voltage of the amplifier drives complementarily connected transistors that altogether form the output power stage. Depending on the sign of voltage u_w only one transistor T_1 or T_2 is conducting and in the secondary circuit a compensating current – i_{2e} is forced (Fig. 2). The flow of current i_{2e} produces an additional magnetic flux ψ_e associated with the secondary winding. The ψ_e flux compensates the main flux that is equal to $\psi_t = z_2(\phi_2 - \phi_1)$, which is generated in the current transformer. Due to the compensation of the magnetic flux in the magnetic core, the primary and secondary flows are equal each to other:

$$i_1 z_1 = i_2 z_2 . \tag{3}$$

In reality, primary and secondary flows in Eq. (3) are only approximately equal because of the control error. Furthermore, to work properly, current transformers need a small residual flux in the magnetic core. The voltage drop that appears on the measurement resistor R_M is the response signal of the electronic current transducer to the primary current i_1, can be calculated from the relation:

$$u_2 = R_M i_1 \frac{z_1}{z_2} \quad . \tag{4}$$

Some parameters of the electronic current transducer: z_1, z_2, R_1, R_2, R_M can be easily found in its specification. The remaining parameters of the model i.e. S_{Fe}, l_{Fe}, l_0, k_H, A_{v0}, ω_g, L_2, L_{2r}, R_{Fe}, B_r, G are unknown to the user. Sometimes if construction of electronic transducer is not hermetically closed, parameters S_{Fe}, l_{Fe}, l_0 can be easily determined.

The identification method of nonlinear objects was used to estimate the values of unknown parameters. It is worth emphasizing that adjustment of all the unknown parameters at the same time is inadvisable as it could cause unsettling of the part of them which also results from Fig. 2. Therefore, it is strongly recommended to accept most of the unknown parameters as constants and in the identification process only those unknown parameters that affect significantly the value of the electronic current transducer output signal – i_2 should be determined.

3. SELECTED METHODS OF IDENTIFICATION OF PARAMETERS OF GENERALLY NON-LINEAR MODELS

At present there are numerous well-known identification methods of parameters of non-linear object models. A thorough description of these methods can be found in the following works [1, 3, 6, 12]. Choosing the method, the authors have taken into consideration not only its usefulness in the parameter identification of the electronic current transducer model, but also possibilities of its implementation in C++ language as well as its effectiveness. Therefore, the lead time of the procedure of finding the parameters of the best fit played the essential role here.

A common part for all methods of the model parameters identification is to define the so-called figure-of-merit function which measures the agreement between the responses of the real object and a model $\Phi(u, \mathbf{a})$, obtained for the same test-signal. The merit function is small when the agreement between the examined object and its mathematical model is good. Then parameters of the model are changed to minimalize the merit function. For measurement points (u_i, y_i) with standard deviation σ_i, $i = 1, 2, \ldots, N$ and the mathematical model $\Phi(u, \mathbf{a})$, the exemplary figure-of-merit function is defined as:

$$F(\mathbf{a}) = \sum_{i=1}^{N} \left(\frac{y_i - \Phi(u_i, \mathbf{a})}{\sigma_i} \right)^2. \tag{5}$$

The Eq. (5) will be realized only when the errors of subsequent measurements of the response of a real object y_i have a normal distribution and are linearly independent. Otherwise Eq. (5) has an approximate character.

It often happens that a merit function has not one, but many local minima. Of course in the process of identification it is essential to find the global minimum, which causes many difficulties, when procedures of searching for best fit parameters encounter local minima.

Moreover, in the process of best fit parameter finding, it is important to take into consideration the fact that the data obtained during laboratory measurement contain random errors [2]. For that reason the constructed model, even if it were most precise and correct, would never ideally imitate the measurement data.

The fitting procedure should provide:

1. parameters of best fit,
2. estimation errors of the found parameters,
3. a statistical measure of goodness of fit.

If the following requirements concerning the implementation of the algorithm were added to the above problems, i.e.:

1. optimization of the program code (assures quick adjustment of the model),
2. correctness with respect to numerical methods (roundoff errors, cutoff errors etc.), the process of finding best fit parameters would become really complicated. Taking that into account, the authors used an already existing procedure of parameter identification [12], modifying its source code and introducing several innovative changes to the algorithm of the procedure.

In identification methods discussed in next subsections the gradient and the Hessian matrix of the merit function are used, therefore their mathematical definitions have been presented.

The gradient of the merit function in relation to parameters **a** is defined as follows:

$$\nabla F(\mathbf{a}) = \frac{\partial F(\mathbf{a})}{\partial \mathrm{a}_k} = -2\sum_{i=1}^{N} \frac{y_i - \Phi(u_i, \mathbf{a})}{\sigma_i^2} \frac{\partial \Phi(u_i, \mathbf{a})}{\partial \mathrm{a}_k}, \tag{6}$$

where: $k = 1, 2, 3, \ldots, M$. The gradient has a zero-value when the merit function reaches a minimum.

Counting partial derivatives after parameters **a** for the merit function (on condition that the merit function is twice differentiable) the Hessian matrix is obtained, the subsequent values of which are given as:

$$\nabla^2 F(\mathbf{a}) = \frac{\partial^2 F(\mathbf{a})}{\partial \mathrm{a}_k \partial \mathrm{a}_l} = 2\sum_{i=1}^{N} \frac{1}{\sigma_i^2} \left[\frac{\partial \Phi(u_i, \mathbf{a})}{\partial \mathrm{a}_k} \frac{\partial \Phi(u_i, \mathbf{a})}{\partial \mathrm{a}_l} - (y_i - \Phi(u_i, \mathbf{a})) \frac{\partial^2 \Phi(u_i, \mathbf{a})}{\partial \mathrm{a}_k \partial \mathrm{a}_l} \right], \tag{7}$$

where: $k = 1, 2, 3, \ldots, M$, $l = 1, 2, 3, \ldots, M$.

In the next subsections the selected methods of parameter identification have been briefly described. The order of the presentation of these methods is not accidental, because Newton's method and the steepest descent method determine the foundation for the Levenberg-Marquardt's method that is the object of interest in this article.

3.1. Newton's method

Newton's method belongs to methods of local convergence. The following conditions must be accomplished to achieve a locally convergent method:

1. the merit function is twice differentiable and in the neighborhood of the solution, given as, $\mathbf{a}^*$ for a certain constant γ the estimation takes place $\|\nabla^2 F(\mathbf{a}') - \nabla^2 F(\mathbf{a}'')\| \le \gamma \|\mathbf{a}' - \mathbf{a}''\|$, where: $\mathbf{a}'$ and $\mathbf{a}''$ are two different vectors of parameters,
2. $\nabla F(\mathbf{a}^*) = 0$,
3. $\nabla^2 F(\mathbf{a}^*)$ is positively defined.

If the above mentioned conditions are realized then the function $F(\mathbf{a})$ could be approximated by function $\tilde{F}(\mathbf{a})$ which is described by a quadratic form:

$$\tilde{F}(\mathbf{a}) = F(\hat{\mathbf{a}}) + \nabla F(\hat{\mathbf{a}})^T (\mathbf{a} - \hat{\mathbf{a}}) + \frac{1}{2} (\mathbf{a} - \hat{\mathbf{a}})^T \nabla^2 F(\hat{\mathbf{a}}) (\mathbf{a} - \hat{\mathbf{a}}), \tag{8}$$

where: $\hat{\mathbf{a}}$ is the vector of parameters in the current iteration, **a** is the vector of parameters in the new iteration, T determines the transposition operation and $\nabla^2 F(\hat{\mathbf{a}})$ is the Hessian matrix of the merit function. The Eq. (8) was introduced by developing the function $F(\mathbf{a})$ into a Taylor series and by rejecting the rest of the Taylor formula.

If $\nabla^2 F(\hat{\mathbf{a}})$ is positively defined, then the minimum of the function $\tilde{F}(\mathbf{a})$ is determined by comparing the gradient of the function to zero, which can be written as:

$$0 = \nabla\tilde{F}(\mathbf{a}) = \nabla F(\hat{\mathbf{a}}) + \nabla^2 F(\hat{\mathbf{a}})(\mathbf{a} - \hat{\mathbf{a}}). \tag{9}$$

After simple operations the vector of parameters is obtained in the new iteration:

$$\mathbf{a} = \hat{\mathbf{a}} - \left(\nabla^2 F(\hat{\mathbf{a}})\right)^{-1} \nabla F(\hat{\mathbf{a}}). \tag{10}$$

If conditions (1–3) are met and the starting point $\mathbf{a}^0$ is found close enough to the solution $\mathbf{a}^*$, then subsequently counted parameters $\mathbf{a}^n$ will approach to $\mathbf{a}^*$, which can be presented as:

$$\mathbf{a}^n = \mathbf{a}^{n-1} - \left(\nabla^2 F\left(\mathbf{a}^{n-1}\right)\right)^{-1} \nabla F\left(\mathbf{a}^{n-1}\right), \tag{11}$$

In practice the inverse matrix to the Hessian matrix is not counted, but the set of equations is solved in relation to $\delta\mathbf{a}$:

$$\nabla^2 F\left(\mathbf{a}^{n-1}\right)\delta\mathbf{a} = -\nabla F\left(\mathbf{a}^{n-1}\right). \tag{12}$$

Then, new parameters are obtained as follows:

$$\mathbf{a}^n = \mathbf{a}^{n-1} + \delta\mathbf{a}. \tag{13}$$

Newton's method is characterized by the local convergence, which means that if the vector of starting parameters is far from the searched solution $\mathbf{a}^*$ then Newton's algorithm will be unstable and will not lead to the desirable solution. In practice at the beginning of the merit function minimum search, the method of global convergence is used. The method is described in the next subsection.

3.2. The steepest descent method

Here the chosen method of global convergence - the steepest descent method is discussed. In the method the values of new parameters are obtained from the equation:

$$\mathbf{a}^n = \mathbf{a}^{n-1} - \lambda\nabla F\left(\mathbf{a}^{n-1}\right). \tag{14}$$

However, as opposed to Newton's method, the new set of parameters $\mathbf{a}^n$ is not always close to the optimum solution $\mathbf{a}^*$.

Different ways of coefficient λ selection are applied to accelerate the calculation of the method. The basic strategy of determining parameter λ is the following: when in the next step of the algorithm the value of the merit function $F(\mathbf{a})$ decreases, the coefficient λ is slightly increased, whereas when the value of the merit function increases significantly in relation to the previous step of the algorithm (probably the solution grows distant from the global minimum of the function $F(\mathbf{a})$), the value of the coefficient λ is decreased. It should be emphasized here that the ways of coefficient λ selection are most often arbitrary. In spite of different ways of coefficient λ selection described in [6], more or less effective, the steepest descent method works slowly (the quantity of iterations which ought to be performed to reach the optimum solution, is large).

In the next subsection still another method will be discussed which has the features of Newton's method and of the steepest descent method, hence it becomes an attractive tool to solve non-linear problems.

3.3. The Levenberg-Marquardt's method (LM)

The authors examine the model parameters estimation method worked out by Levenberg and Marquardt.

For the merit function given with the Eq. (5), the vector of parameters **a** determined in n-th iteration of the LM method defines the equation [6, 3]:

$$\mathbf{a}^{n} = \mathbf{a}^{n-1} - \left(\boldsymbol{H} + \lambda\mathbf{1}\right)^{-1} \nabla F\left(\mathbf{a}^{n-1}\right), \tag{15}$$

where: **1** is the singular matrix of the dimension M, $\boldsymbol{H}$ is the Hessian matrix of the merit function, also of the dimension M.

If λ parameter approaches zero, the way of determining the successive approximations of the vector of parameters **a** will be close to Newton's method. However, the increase of parameter λ causes an increase of importance of the direction of improvement which is determined on the basis of the gradient of the merit function $F(\mathbf{a})$. The λ parameter is not only used as „a switch" between two strategies of searching for the optimum solution i.e. with Newton's method and the steepest descent method, but it also affects the step size performed towards the minimum of the function $F(\mathbf{a})$. It turns out that the greater the λ coefficient, the smaller the value of the step. These features of the LM method are very useful, because at the beginning, when the considered parameters are distant from the optimum solution, then the performed steps are smaller and the steepest descent method is used. Newton's method is used only when the algorithm is close to the solution, which assures greater stability of the LM method.

The set of linear equations is solved in n-th step of the LM method in the form [12]:

$$\sum_{i=1}^{M} \alpha'_{kl} \delta a_l = \beta_k, \tag{16}$$

where: $\begin{cases} \alpha'_{kl} = \alpha_{kl}(1+\lambda), \text{dla } k = l \\ \alpha'_{kl} = \alpha_{kl}, \text{dla } k \neq l \end{cases}$

and

$$\beta_k = -\frac{1}{2}\frac{\partial F(\mathbf{a})}{\partial a_k}, \tag{17}$$

$$\alpha_{kl} = \frac{1}{2}\frac{\partial^2 F(\mathbf{a})}{\partial a_k \partial a_l}. \tag{18}$$

The LM method steps are performed as follows:

a) Assume a priori the set of trial parameters $\mathbf{a}^0$,

b) Calculate the value of the merit function for trial parameters $F(\mathbf{a}^0)$,

c) Accept a small value of the parameter λ, for example $\lambda = 0{,}001$,

d) Solve the linear set of Eq. (16) in relation to $\delta\mathbf{a}$ and then calculate the value of the function $F(\mathbf{a} + \delta\mathbf{a})$,

e) If $F(\mathbf{a} + \delta\mathbf{a}) \geq F(\mathbf{a})$ is true, then increase the parameter λ 10-times (another scaling coefficient can also be used) and return to step d),

f) If $F(\mathbf{a} + \delta\mathbf{a}) < F(\mathbf{a})$ is true, then decrease the parameter λ 10-times, then update the vector of parameters according to the equation $\mathbf{a} = \mathbf{a} + \delta\mathbf{a}$ and go to step d).

Large demand for the memory which is proportional to the square of the number of adjusted parameters **a** is a disadvantage of the LM algorithm. Therefore, the application of this algorithm in models with a big number of parameters is inadvisable.

The model considered in this paper has a small number of parameters, that is why, in the view of numerous advantages of the method i.e. stability and the comparatively quick convergence, the authors have decided to choose the LM method for the identification of the parameters of the electronic current transducer model.

4. RESULTS OF PARAMETER IDENTIFICATION OF THE ELECTRONIC CURRENT TRANSDUCER

Using procedures described in work [12] and introducing own modifications during the implementation of the described LM method, the authors of the report have created the application for calculating parameters of the electronic current transducer model.

As the mathematical model is given in the form of ordinary differential equations [11] of the first-order (the model is not given transparently in the form of a function $\Phi(u, \mathbf{a})$), therefore the authors had to modify the procedures of the program, among others for calculating the gradient. The problem „of the dead end", when the algorithm originally written stopped in local minima, has also been solved.

Figure 3 presents a graphic window of the program that represents the beginning of the identification process, when the parameters are still distant from their optimum values. The procedure of the identification has been realized for a transducer of the type LA25-NP [13].

In the program are combined amplitude characteristics of the real object (dots) and mathematical model (solid line). Manufacturers assess the cutoff frequency based on amplitude characteristics for sinusoidal primary current [5, 8, 9].

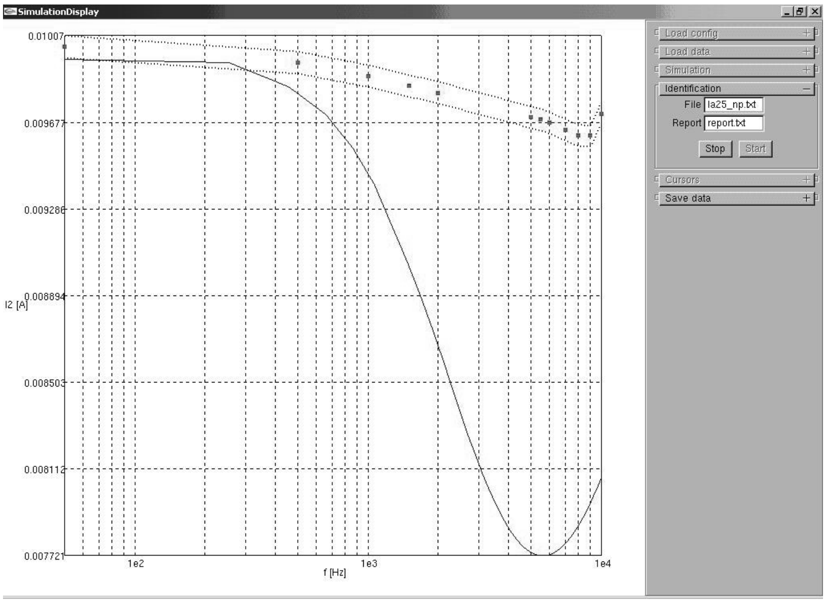

Fig. 3. Bode characteristics of electronic current transducer obtained for the first trial values of the parameters.

In the panel "Identification" there is given "a teaching file", which consists of a series of measurement results of the secondary current effective value of the electronic current transducer performed for several selected values of the frequency of the primary current of settled effective value. The information which parameters have to be adjusted, and which remain constant are found in the configuration file read in from the panel "Load config" of the main program. Fragment of the exemplary configuration file is listed below:

```
# [V/T] the constant of the Hall effect
kH    2.75  "var"
# [V/V] DC amplification coefficient
Av0   100e3 "const"
# [Hz] 3dB-cutoff angular frequency of the amplifier
wg    1.1e6 "const"
```

The configuration file also contains: parameters of the differentiation method settings, the arrangement of the graph and parameters of the test-signal that is given at the entry of the model. Changes of the test signal are performed during simulation (the panel "Simulation"), when the parameter identification has been completed. During identification (the panel "Identification") the sinusoidal signal is used with the effective value equal to the rated value of the primary current read from the specification of the examined transducer. Measurements for the teaching file were also performed for the same effective value of the primary current for a dozen or so frequencies of the values of the transformation bandwidth of the electronic transducer block. The results of the identification along with the errors of the parameter estimations in each iteration are printed in the report file.

The description of other panels is the following: "Load data" - showing the measurements obtained during the simulation, "Cursors" - the use of cursors for precise determination of the value of the cutoff frequency, "Save" - recording of the image of the graphic window into a BMP file.

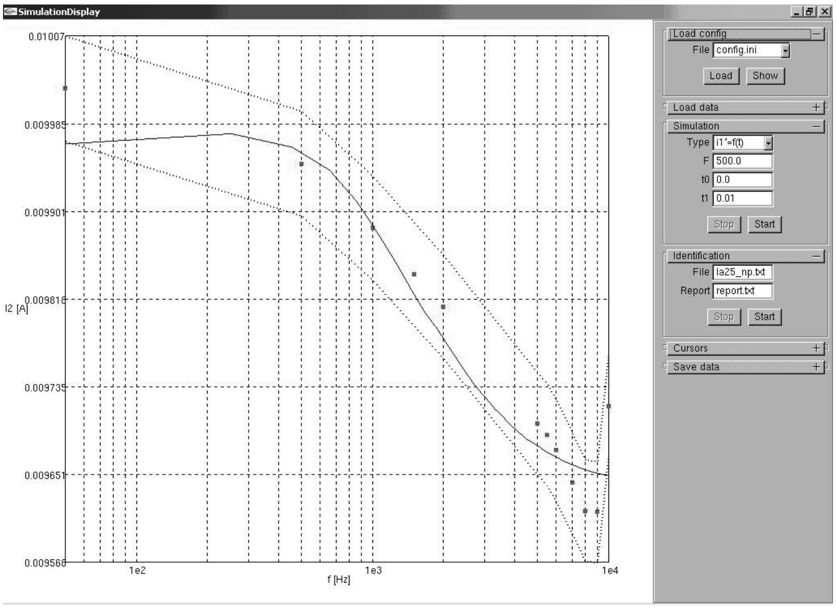

Fig. 4. Bode characteristics of electronic current transducer obtained for extracted values of the parameters.

The limiting error of the LA 25-NP transducer is 0,5 % of the measured current. Taking into account the limiting error value, characteristics have been drawn of the error of the secondary current effective value for the real transducer (the characteristics of limiting errors are marked with dashed lines in Fig. 3 and Fig. 4). The result of the simulation of the model with identified parameters are marked with a solid line. Points represent measurements performed for the teaching file, on the basis of which the identification has been performed. In Table 1 were presented values obtained from specification: z_1, z_2, R_1, R_2, R_M and values: l_{Fe}, l_0, S_{Fe} measured

Table 1.

Parameter	Unit	Value
z_1	-	5
z_2	-	1000
R_1	Ω	0.00125
R_2	Ω	110
R_M	Ω	190
l_{Fe}	m	0.053
l_0	m	1e-3
S_{Fe}	m^2	8.625e-6

based on LA25-NP transducer. In Table 2 were presented values for which identification process was performed. Values from Table 1 were set as constants in configuration file and values in Table 2 as variables.

Table 2.

Parameter	Unit	First trial value	Extracted value
k_H	V/T	5.00	2.75
A_{v0}	V/V	100e3	103e3
ω_g	rad/s	6.28e6	7.41e6
L_2	H	0.02	0.0249
L_{2r}	H	2e-5	1.69e-4
R_{Fe}	Ω	1e5	1.21e6
B_r	T	1e-6	0.12e-6
G	S	0.0125	0.0132

The final result of the identification which was realized in seven iterations is presented in Fig. 4 and Table 2. The result of the identification is within the error limit of the measurement of the secondary current effective value of the transducer.

5. CONCLUSION

The method of parameter identification of the electronic current transducer model has been discussed in this paper. The method appeared to be an efficient tool for the determination of unknown parameters of the transducer model, because the answers of the model of parameters identified with the LM method are within the error limit of the measurement with the electronic current transducer.

The new model with well identified parameters can contribute to the recognition of the metrological proprieties of electronic transducers and to the qualification of the usefulness of these transducers in processing of a deformed signal. Other advantages, which result from the application of the model are: the possibility of the use of the model during research in working conditions different from normal and choosing the electronic transducer which would be optimal in a given measurement system.

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