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Zbigniew Kłos, Jarosław Krysiak

Wroclaw University of Technology
Institute of Electrical Machines, Drives and Metrology
Poland, e-mail: zbigniew.klos@pwr.wroc.pl

FACTORS LIMITING EXTREMELY SENSITIVE MEASURING RANGES OF CONTEMPORARY PICOAMMETERS

In the paper the principle of operation of contemporary picoammeters with analog conversion of current into voltage has been presented. The influence of factors disturbing the i/u conversion in a passive as well as in an active converter has been analyzed.

It has been shown that a picoammeter with the most sensitive ranges should base on active i/u conversion. In detailed considerations maximal errors coming from internal sources (i.e. input bias current, offset, noise currents and noise voltages of amplifier and thermal noise of a high-ohm scaling resistor) as well as external disturbances (leakage currents of printed board and insulators and disturbances in leading cables) have been discussed and evaluated. From this analysis it can be concluded that on the extremely sensitive ranges the most leading impact on the accuracy of the current measurement have the following factors: the input bias current of the electrometric amplifier, accuracy of the scaling resistor and its thermal noise. These factors limit the extremely sensitive ranges of picoammeter to the level of 10÷20 pA with an accuracy of 1.2%. The presented considerations can be helpful for designers as well as for users of extremely sensitive picoammeters because they permit to avoid some errors caused by improper configuration of the measuring circuit and to distinguish disturbances coming from different sources.

Keywords: active i/u conversion, operational amplifier, scaling resistor

1. INTRODUCTION

The progress which took place in technology of monolithic amplifier with input FET-transistors [1] caused that parameters of these amplifiers do not retreat parameters of DC amplifiers with vibration capacitor processing. This concerns especially the input current of FET-transistors which has been reduced to a level of 10÷50 fA.

Thanks to the development of a new generation of precise high-ohm metal-oxide resistors [2, 3], the parameters of these elements are considerably improved. It implies that measurement possibilities of picoammeters using such elements (i.e. electrometric operational amplifiers and high-ohm metal-oxide resistors) reach the boundary of measurability of extremely small DC and low-frequency currents limited by present-state technology in the world. Picoammeters became cheaper and simpler in operation and their low ranges reached the level of pA [4]. This implies the growth of interest about such instruments and their wide use in scientific research, environment protection, technology and industry.

Measurements and continuous registration (monitoring) of low DC currents and low-frequency signals are used in some areas of scientific investigation, particularly in material engineering, biophysics, biomedicine, chromatography, measurement of ions etc. Registration of changes in conductivity currents of insulating materials and the high-resistance layers measured at low voltages (material engineering), registration of ion current in stimulated membranes of biological vegetable cells (biophysics) and live human cells (medicine) require often measuring systems with the most sensitive measuring ranges [5]. Similarly, highly sensitive picoammeters are used in vacuum-meters for monitoring the state of vacuum in some technological processes (production the integrated circuit) or in on-line control of chemical constitution (using a chromatograph) of fuel gas delivered for industrial and domestic use.

The presented application domains of electrometric techniques of measurement of extremely low DC and low-frequency currents show that the contemporary picoammeters have to make possible not only the measurement of the current but simultaneously registration (monitoring) of temporary variation of current values. Such requirements decide that the measurement of the current has to be based on analog i/u conversion, what determines the principle of operation of the instruments considered. For this reason almost all picoammeters manufactured in the world use analog i/u converters [6] (passive as well as active) as main blocks. In the article a comparison of both types of analog i/u conversion mentioned will be presented. Moreover, the factors limiting the most sensitive ranges of measured current and the accuracy of measurement will be discussed.

2. THE PRINCIPLE OF OPERATION OF A PICOAMMETER WITH ANALOG I/U CONVERSION

A block diagram of a typical picoammeter with an analog i/u converter (passive or active) is presented in Fig. 1.

The picoammeter consists of two blocks: the analog i/u conversion block and the current reading block. The basic block of the picoammeter, deciding about its measurement parameters, ranges and accuracy – is the input block of analog i/u conversion. Depending on the i/u converter used, two main types of blocks of analog conversion can be distinguished:

- Block of active i/u conversion. It consists of two stages: a proper active i/u converter and a normalizing amplifier.
- Block of passive i/u conversion. It consists of three stages: a passive i/u converter, a separating buffer and a normalizing amplifier.

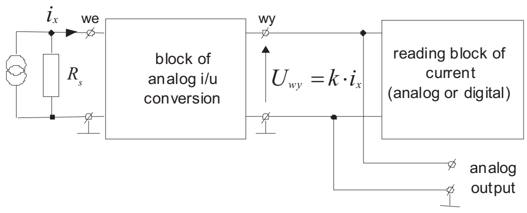

Fig. 1. Block diagram of a picoammeter with analog i/u conversion.

The measurement of low current is possible only after conversion of this quantity into voltage at a suitably-high level at which the influence of disturbing factors is already small. The output voltage from the analog i/u conversion block is proportional to the measured current i_x. Next, this voltage is passed on to the input of the of the current reading block, which is an analog or digital voltmeter with a range equal to the maximal range of the output voltage U_{wy} of the analog i/u conversion block. The output voltmeter is scaled in units of current. Most of modern picoammeters are provided with digital readout, what considerably improves the reading accuracy.

Moreover, the output voltage U_{wy} from the analog i/u conversion block is led out to auxiliary output terminals, the so-called "analog output", where it is possible to connect an external voltmeter or recorder. Additional "analog output" enables the simultaneous measurement and the registration (monitoring) of momentary changes of the measured current i_x. All contemporary picoammeters have such an auxiliary output which considerably expands their functionality.

3. ANALOG I/U CONVERSION

3.1. Methods of analog i/u processing

The analog conversion of measured current i_x into voltage is possible in one of two types of i/u converters [7], as shown in Fig. 2.

The simplest i/u converter can be a standard high-ohm scaling resistor R_p with an appropriately selected value of resistance. Such a resistor is called a passive i/u converter. If the resistor R_p is included in the negative feedback-loop of an electrometric operational amplifier, then such a converter is called an active i/u converter.
The equation of conversion for an unloaded passive converter is:

$$U_{wy_{i/u}} = i_x R_p = k i_x. \tag{1}$$

The conversion factor k is expressed in units of resistance and has the same value as the scaling resistor i.e. $k = R_p$.

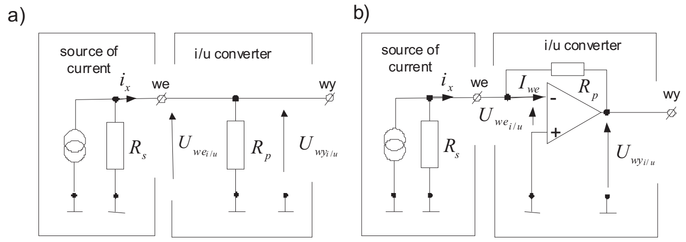

Fig. 2. Circuits of analog i/u conversion: a) passive i/u converter, b) active i/u converter.

Assuming that the operational amplifier is ideal ($I_{we} = 0$, $k_{u0} = \infty$), the equation of conversion of the active i/u converter can be expressed as follows:

$$U_{wy_{i/u}} = -i_x R_p = k i_x. \quad (2)$$

The conversion factor k is equal to $k = -R_p$ in the active i/u converter. It can be shown that the polarization of the output voltage of the converter is inverted in relation to the polarization of the measured current i_x, which requires the use of an additional inverting stage.

The i/u conversion should be made with a possibly small value of resistance R_p, because using a high-ohm scaling resistor implies deterioration of its parameters. It means that the output voltage, which the measured current is converted to, is not very high for low currents. The value of the output voltage cannot be very low, because the influence of disturbing effects, i.e. noise, is especially meaningful at small values of the output voltage. As a trade-off between the mentioned conditions, the output voltage of the analog i/u converter is assumed to reach a level of 0.1÷1 V at the end of each current range of the picoammeter.

3.2. Parameters of analog i/u converters

Input resistance of i/u converters

Passive i/u converter

As shown in Fig. 2a, the input resistance of the passive i/u converter is equal to the resistance of the scaling resistor

$$R_{we_{i/u}} = R_p. \quad (3)$$

The resistance of the scaling resistor R_p on the considered range of the picoammeter depends on the maximal output voltage of the converter occurring at the maximal measured current on this range

$$R_p = \frac{U_{wy_{\max}}}{I_z}, \tag{4}$$

where: $U_{wy_{\max}}$ – maximal output voltage of the converter, I_z – maximal current on the considered range of the picoammeter.

Taking into account the influence of disturbances occuring during i/u processing, the output voltage at the end of the current range should not be lower than 0.1÷1 V. Then for the processing of low currents, e.g. 1 pA, the scaling resistor R_p with the resistance value obtained from the Eq. (4), at the assumed voltage $U_{wy_{i/u}}$ = 0.1÷1 V, is equal to 10^{11}÷10^{12} Ω, and so it is also very high. Such a situation is very disadvantageous and it is a reason for the appearance of considerable systematic errors.

Active i/u converter

The input resistance of the active i/u converter can be described by the expression [6, 8]

$$R_{we_{i/u}} = \frac{R_p}{k_{u0}}, \tag{5}$$

where: k_{u0} – open-loop gain of the operational amplifier presented in Fig. 2b, R_p – resistance of the scaling resistor.

Unlike the previously presented passive i/u converter, the input resistance of the active i/u converter is essentially smaller (theoretically close to zero because $k_{u0} \to \infty$). It is a very important advantage of the active i/u converter because the small value of the input resistance of this converter (being in fact the input resistance of the whole picoammeter) does not generate a big systematic error after connecting the picoammeter into the current measurement circuit. It implies that the value of the scaling resistor R_p is practically not limited, and, consequently, the lowest range of the picoammeter can be substantially lower (more sensitive).

Output resistance of i/u converters

Passive i/u converter

The output resistance as well as the input resistance of the passive converter is equal to the value of the scaling resistor

$$R_{wy_{i/u}} = R_p. \tag{6}$$

For the previously assumed most sensitive range, e.g. 1 pA, the value of the output resistance equals 10^{11}÷10^{12} Ω. A high value of the output resistance $R_{wy_{i/u}}$ of the i/u passive converter creates difficulties in the measurement of the output voltage $U_{wy_{i/u}}$ of the converter. In such a situation it is necessary to use in the next stage of the picoammeter a special amplifier, which features a very high input resistance (many times higher than the output resistance of the passive i/u converter).

Active i/u converter

The output resistance of the active i/u converter is also the output resistance of the operational amplifier W1. It is defined due to [8] by the parameters of the amplifier

$$R_{wy_{i/u}} = \frac{k_{uf}}{k_{u0}} R_{wy0}, \tag{7}$$

where: R_{wy0} – output resistance of the operational amplifier presented in Fig. 2b with the open loop (usually 100÷200 Ω), k_{u0} – open-loop gain of the operational amplifier (usually $k_{u0} \approx 10^5$), k_{uf} – feedback-loop gain of the operational amplifier (usually $k_{uf} \leq 2$).

Unlike the passive i/u converter, the active i/u converter has the output resistance depending only on parameters of the operational amplifier, as shown in (6), not on the scaling resistor. In typical operational amplifiers with a closed feedback loop the output resistance is very small, which has positive consequences in practice. It means that overloading of the output of the i/u converter by the input resistance of the next stage of the picoammeter does not disturb the i/u conversion process.

Input voltage of i/u converters

Passive i/u converter

The input voltage of the passive i/u converter equals the voltage drop on the scaling resistor R_p

$$U_{we_{i/u}} = U_{R_p}. \tag{8}$$

The input voltage $U_{we_{i/u}}$ of the passive i/u converter is the same as its output voltage $U_{wy_{i/u}}$. As mentioned earlier, this voltage is rather high, usually 0.1÷1 V at the end of each current range.

Active i/u converter

Differently than in the case of the passive i/u converter, the input voltage of an active i/u converter is determined by the input voltage of the used operational amplifier. This voltage depends on the output voltage of the operational amplifier and its coefficient k_{u0} due to [6, 8] in the following way:

$$U_{we_{i/u}} = \frac{U_{wy}}{k_{u0}}. \tag{9}$$

Because the output voltage of the amplifier is in fact the output voltage of the converter, then, for the assumed maximal voltage $U_{wy_{i/u}} \leq (0.1 \div 1)$ V and $k_{u0} = 10^5$, the voltage occurring at the input of the converter is not higher than 1÷10 μV. This feature is especially useful in the measurement of high resistance using the indirect method, if the testing voltage is low. In such a situation the voltage drop across the input of the picoammeter can be neglected and then almost the full voltage appears on the measured resistor [5].

It is worth to mention that the metrological parameters, i.e. input resistance $R_{we_{i/u}}$, output resistance $R_{wy_{i/u}}$ and input voltage $U_{we_{i/u}}$, are completely different in both types of converters. These problems will become exposed in further considerations.

4. PICOAMMETER WITH PASSIVE I/U CONVERSION

4.1. The principle of operation

A simplified diagram of a picoammeter with passive i/u converter working in real conditions is presented in Fig. 3.

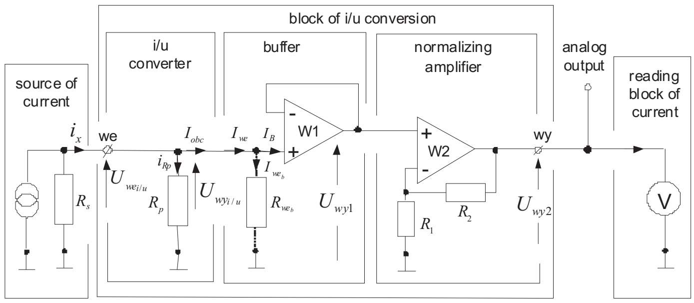

Fig. 3. Schematic diagram of a picoammeter with a passive i/u converter.

The presented picoammeter includes two main blocks: a passive i/u conversion block and a current reading block (voltmeter). The i/u conversion block consists of three stages: an input passive i/u converter, a separating buffer and a normalizing amplifier. The passive i/u converter transforms the measured current i_x into the output voltage

$$U_{wy_{i/u}} = i_x R_p. \tag{10}$$

This voltage is the input voltage to the next stages, e.g. a separating buffer with high input resistance. Usually this role is played by an operational amplifier. Such a circuit does not change the polarization of the voltage signal and makes possible the measurement of the output voltage coming from the i/u passive converter without loading this output

$$U_{wy1} = k_{u1} U_{wy_{i/u}}, \tag{11}$$

where: U_{wy1} – output voltage of the buffer amplifier W1, k_{u1} – voltage gain coefficient of the buffer amplifier W1.

The non-inverting normalizing amplifier W2 with a voltage gain coefficient equal to

$$k_{u2} = 1 + \frac{R_2}{R_1}, \tag{12}$$

adapts the value of the output voltage from the i/u conversion block to the range of the final voltmeter used for the readout of the current. The voltage at the output of the normalizing amplifier can be expressed as

$$U_{wy2} = k_{u2} U_{wy1} = k_{u2} k_{u1} R_p i_x = k i_x. \tag{13}$$

Note that the conversion factor k is the product of the voltage gain coefficients k_{u1} and k_{u2} of amplifiers W1 and W2, respectively, and the resistance of the scaling resistor R_p.

4.2. Factors limiting the lowest range of the picoammeter with passive i/u conversion

The main limitation of the most sensitive ranges of the considered picoammeter is the very high input and output resistance of the passive i/u converter discussed in section 3.2. An additional source of errors can be the operational amplifier used as the separating buffer. In further considerations we will describe errors generated by the mentioned two sources of disturbances in the picoammeter with a passive i/u converter.

Influence of parameters of the passive i/u converter

Input resistance of the converter

The input resistance of the passive i/u converter is equal to the resistance of the scaling resistor R_p and at the same time it is the input resistance R_{pA} of the picoammeter. On the most sensitive ranges of the instrument the value of the resistance R_{pA} is very high, what implies that after connecting the picoammeter to the current measurement circuit with the internal resistance R_s, a change of the measured current i_x is noticeable. It causes that some relative systematic error δ_m appears

$$\delta_m = -\frac{R_{pA}}{R_{pA} + R_s}. \tag{14}$$

For proper measurement of the current it is necessary to preserve a condition $R_{pA} << R_s$. If such an assumption is fulfilled then the Eq. (14) reduces to the rough formula

$$\delta_m \cong -\frac{R_{pA}}{R_s}. \tag{15}$$

If the source of the measured current does not have a very high internal resistance R_s, then the systematic error δ_m can be so large that the measurement is practically impossible. For example, if the error δ_m should be not greater than 1%, then the internal resistance $R_S$has to be about 100 times larger than the input resistance R_{pA} of the picoammeter.

As we have mentioned in section 3.2, on the 1 pA range the input resistance $R_{pA} = R_p$ is achieving a considerably high value, even $10^{11}\div10^{12}$ Ω. In such a case, the internal resistance of the source of the measured current should be higher than $10^{13}\div10^{14}$ Ω. Unfortunately, current sources with such a big internal resistance are extremely rare. In practice the measured current i_x comes from real objects with unpredictable internal resistance, which usually is not higher than $10^{8}\div10^{9}$ Ω. For such sources of current the resistance R_p should be not higher than 1÷10 MΩ, which is equivalent to the current range of the picoammeter equal to 10÷100 nA, sometimes 1÷2 nA. It is a basic limitation in the use of picoammeters with passive i/u conversion.

Output resistance of the converter

A high value of the output resistance $R_{wy_{i/u}}$ of the passive i/u converter creates difficulties in the measurement of the output voltage $U_{wy_{i/u}}$ of the converter. It is a consequence of the fact that after connecting the buffer amplifier to the output of the converter, the input resistance of the buffer R_{we_b} (presented in Fig. 3) is a by-pass for the scaling resistor R_p. It implies the necessity of applying of a special electrometric amplifier (with very high input resistance and ultra-low input current) as a buffer. The relative systematic error δ_{R_b} triggered by this phenomenon can be expressed by the formula

$$\delta_{R_b} = -\frac{R_p}{R_p + R_{we_b}} \cong -\frac{R_p}{R_{we_b}}. \tag{16}$$

Minimization of this error to the level of 0.1% on the 1 pA range ($R_p = 10^{11}\div10^{12}$ Ω) requires the use of a buffer amplifier with an input resistance of at least $10^{14}\div10^{15}$ Ω. On the higher ranges this condition is less restrictive (easier to fulfill). Obtaining such a big input resistance is possible only in special electrometric amplifiers.

Output voltage of the converter

The output voltage $U_{wy_{i/u}}$ of the passive i/u converter is rather high, usually 0.1÷1 V at the end of each current range. The high value of $U_{wy_{i/u}}$ implies an increase of the input current of the next stage, which is a separating buffer.

Influence of the parameters of the buffer amplifier

Input resistance of the buffer amplifier

Discussion of the systematic error δ_m defined by (15) has shown that in measurements of current coming from practical sources the value of the scaling resistor can not be very high, i.e. $R_p \leq 1 \div 10$ MΩ. It is necessary to use an operational amplifier with an input resistance $10^9 \div 10^{10}$ Ω as a buffer. Most of contemporarily produced monolithic operational amplifiers, especially with input FET transistors, has input resistance high enough to fulfill conditions resulting from Eq. (15).

Input current of the buffer stage

As has been mentioned earlier, the input current I_{we} of the buffer is loading the converter's output, therefore it changes the value of the current passing through the standard scaling resistor R_p

$$i_{R_p} = i_x - I_{we}. \tag{17}$$

It implies that the voltage drop $U_{R_p} = i_{R_p} R_p$ changes. The systematic error caused by this effect can be described by the expression

$$\delta_{I_{web}} \cong -\frac{I_{we}}{i_x}. \tag{18}$$

The input current of the operational amplifier I_{we} is the sum of two elements: the input bias current I_B and the proportional component, I_{we_b} as follows:

$$I_{we} = I_B + I_{we_b} = I_B + \frac{U_{wy_{i/u}}}{R_{we_b}}. \tag{19}$$

The input bias current I_B is constant and it does not depend on the input voltage of the amplifier. It is a dominant component. However, if the output voltage of the converter, equal to 0.1÷1 V, is applied to the input of the buffer, then a share of the proportional component depending on $U_{wy_{i/u}}$ increases noticeably. For example, for the amplifier with an input resistance of $R_{we_b} = 10^{14} \div 10^{15}$ Ω, which is necessary on the 1 pA range, the proportional component in the input current is equal to 0.1÷10 fA. It implies an additional systematic error with a value of 0.01÷1%.

On the most sensitive ranges of the picoammeter it is necessary to use an electrometric buffer amplifier with possibly low input current. The best produced operational amplifiers, e.g. OPA-128 of the Burr-Brown company, have an input bias current of $I_B = 10 \div 40$ fA [1], which causes an systematic error equal to $1 \div 4\%$ on the 1 pA range, however on the 1 nA range this error is equal to only $0.001 \div 0.004\%$. It means that the input bias current of the buffer limits in an essential way the lowest range of the picoammeter to the level of $10 \div 100$ pA, if the accuracy of the instrument should be high.

Another solution rarely appearing in picoammeters is the use of a vibration capacitor electrometer as a buffer. The input current of such electrometer is even smaller, equal to $0.1 \div 1$ fA. Unfortunately, the vibration capacitor electrometers are very expensive, difficult in exploitation and moreover sensitive to mechanical shocks and air humidity.

4.3. Summary

The most important disadvantages of the passive i/u converter are high input and output resistance and high input and output voltage $0.1 \div 1$ V. These features limit the value of the scaling resistor R_p, processing i/u in the converter, to the level of $1 \div 10$ MΩ. Such a small value of R_p implies that the most sensitive range of the picoammeter is not lower than $0.1 \div 1$ μA, sometimes $1 \div 2$ nA, although the used operational amplifier has an ultra low input bias current. It means that in a picoammeter with a passive i/u converter it is impossible to fully exploit all modern elements (monolithic electrometric amplifiers and precise high-ohm resistors) which have extremely good metrological parameters. This is why the most sensitive contemporarily manufactured picoammeters with analog i/u conversion in the operation method always use active i/u conversion. Such a picoammeter will be presented in the next section.

5. PICOAMMETER WITH ACTIVE I/U CONVERSION

5.1. The principle of operation

The block scheme of the active i/u converter [7] is presented in Fig. 4.

The picoammeter presented in Fig. 4 consists of two main blocks: an active i/u conversion block and a current reading block. The i/u conversion block has two stages: an active i/u converter and an inverting normalizing amplifier W2.

The measured current i_x is led to the input of electrometric operational amplifier W1, which features an ultra-low input current. Next this signal is converted into the voltage drop $U_{wy_{i/u}}$ on the high-ohm scaling resistor R_p included in the negative feedback-loop of amplifier W1. The voltage $U_{wy_{i/u}}$ appearing at the output of the amplifier W1 with negative polarization is equal to

$$U_{wy_{i/u}} = -(i_x - I_{we})R_p. \tag{20}$$

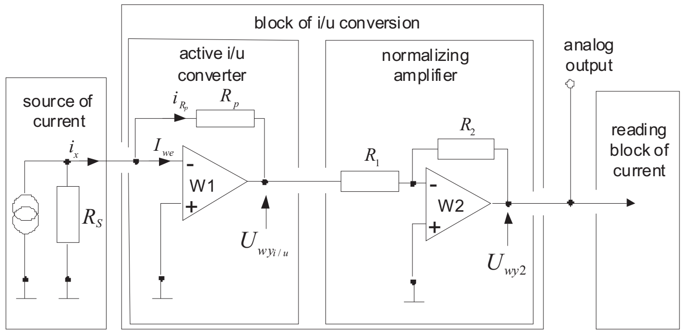

Fig. 4. Schematic of the picoammeter with an active i/u converter.

If the input current of the amplifier is ultra-low ($I_{we} << i_x$), then the equation can be simplified to the relation

$$U_{wy_{i/u}} \cong -i_x R_p. \tag{21}$$

In the normalizing amplifier W2 the polarization of this voltage receives a reverse sign and it is amplified (with an amplification factor $k_{u2} = -(1 \div 10)$) to the level of 1 V at the end of each current range, due to the expression

$$U_{wy2} = -k_{u2} U_{wy_{i/u}} = k_{u2} R_p i_x = k i_x, \tag{22}$$

where: U_{wy2} – output voltage of the normalizing amplifier, $k_{u2} = \frac{R_2}{R_1}$ – absolute value of the voltage gain coefficient of the normalizing amplifier W2, $U_{wy_{i/u}}$ – output voltage of the active i/u converter, R_p – resistance of the scaling resistor on the considered range, i_x – the measured current, k – conversion factor of the active i/u converter.

The voltage U_{wy2} is applied to the "analog output" terminals as well as to the current reading block (analog or digital), where the measured value i_x of the current is obtained from the converted Eq. (22) as follows:

$$i_x = \frac{U_{wy2}}{k_{u2} R_p}. \tag{23}$$

Parameters of the picoammeter with active i/u conversion depend, first of all, on parameters of the first block, especially on input active i/u converter built with an electrometric operational amplifier W1.

5.2. Fitting of the picoammeter with active i/u converter to the current measurement circuit

Amplifying of undesirable internal voltage signals

In the picoammeter with passive i/u conversion, the most important criterion for fitting the instrument to the source of measured current is minimization of the systematic error due to (15). In turn, in the picoammeter with an active i/u converter this criterion is not as restrictive because the input resistance of such picoammeter expressed by the Eq. (5) is relatively small, in many cases negligible.

In the picoammeter with an active i/u converter the most significant effect is amplification of the undesirable voltage signals (noise, offset) coming from amplifier W1. The amplification of disturbing signals depends above all on the internal resistance R_s of the source of measured current and the resistance of the scaling resistor R_p included in the feedback-loop of the amplifier W1. This voltage gain coefficient can be expressed due to [6, 8] as follows:

$$k_{u1} = 1 + \frac{R_p}{R_s}. \tag{24}$$

For the ideal current source ($R_s = \infty$) the gain $k_{u1} = 1$. If the resistance of the real current source is relatively small, e.g. $R_s \leq 10^8\ \Omega$, then on the most sensitive ranges of the picoammeter, where the scaling resistor has a high value $R_p = 10^{11} \div 10^{12}\ \Omega$, the voltage gain of amplifier W1 used in the i/u converter becomes high, usually $10^3 \div 10^4$. In such a case all undesirable voltage signals will be amplified with a large gain, generating considerable disturbances of the measured signal appearing at the output of the i/u converter. Sometimes it can make the measurement impossible and the appearing symptoms can be interpreted as damage of the device. To avoid this problem in practice, the following condition guaranteeing the limitation of the gain to a sensible value, has to be fulfilled:

$$\frac{R_p}{R_s} \leq 1. \tag{25}$$

In extreme cases it is allowed to reach $R_p/R_s = 10$ but then disturbances are really great. Unfortunately, the user usually does not know the precise value of the scaling resistor R_p on any range of the picoammeter. In practical applications, the value of R_p can be evaluated as follows:

$$R_p \leq \frac{U_{wy_{\max}}}{|k_{u2}|\, I_z}, \tag{26}$$

where: $U_{wy_{\max}}$ – output voltage of the i/u conversion block occurring at the maximal current of the range, I_z – maximal current of the selected range of the picoammeter,

$|k_{u2}|$ – absolute value of the voltage gain of the normalizing amplifier W2 (usually $k_{u2} = 1$ for more safe evaluation).

The voltage $U_{wy_{\max}}$ is defined by the manufacturer of the instrument as the maximal voltage admissible at the "analog output" terminals.

Amplifying the internal disturbances in the active i/u converter is very undesirable and this phenomenon can be a source of many unpredictable effects, especially by improper connecting of the instrument to the source of the measured current.

Input current

The input current of the picoammeter with an active i/u converter depends only on the input current of the operational amplifier W1 and is added directly to the measured current i_x at the input of the converter. The systematic error caused by this effect is defined by the relation (18). A detailed explanation of the influence of the amplifier's input current and other factors generating disturbing signals in the active i/u converter will be presented in the next section.

5.3. Summary

The presented analysis of the picoammeter with active i/u conversion shows that this instrument does not have many shortcomings featuring the picoammeter with a passive i/u converter. Its metrological parameters are close to the parameters of an ideal picoammeter: small input resistance and very low input voltage. It is worth to emphasize that the resistance of the scaling resistor R_p is not restricted by the condition of minimizing the input resistance of the picoammeter. As a consequence, the only problem appearing by the use of the picoammeter with an active converter is the fact that by improper configuration of the measured circuit (i.e. $R_s << R_p$) the amplifier W1 used in this converter amplifies the parasitic signals – noise, disturbances, offset, etc. However the advantages are predominant in comparison with the shortcomings of the picoammeter with an active i/u converter and cause that only in picoammeters with active i/u conversion it is possible to achieve extremely sensitive measuring ranges.

6. LIMITATIONS OF THE LOWEST MEASURING RANGES OF THE PICOAMMETER WITH ACTIVE I/U CONVERSION

6.1. Sources of disturbances in the picoammeter with an active i/u converter

In the most sensitive picoammeters as a principle of operation the analog active i/u conversion is applied. As has been presented in section 5, parameters of the active i/u converter determine the possible ranges and accuracy of the device. The most sensitive range of the mentioned picoammeter depends on the minimal value of the range current, which can be transformed into the standard voltage 0.1÷1 V at the end of the range with acceptable accuracy. An attempt to determine this limit requires detecting all factors and disturbances which influence the i/u conversion process. Disturbing signals occur in the electrometric amplifier, scaling resistor R_p, circuit of the measured current, structural elements of the picoammeter and cables. A scheme of the picoammeter with an active i/u converter with indicated sources of the most important disturbing signals [8] is presented in Fig. 5.

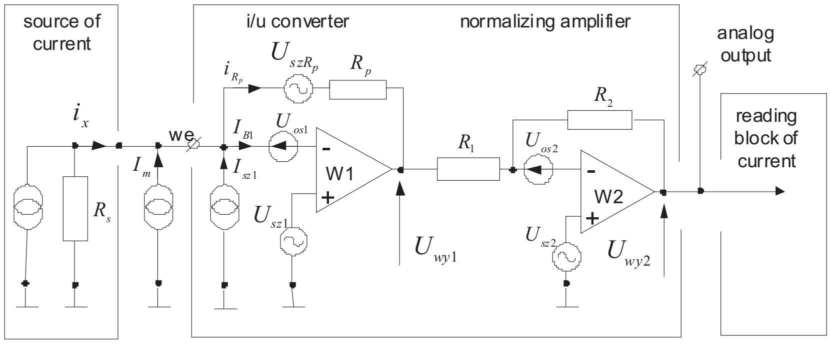

Fig. 5. Scheme of the picoammeter with an active i/u converter and with the most important sources of disturbances pointed out.

If we take into consideration disturbances presented in Fig. 5, the output voltage of the first stage – active i/u converter – can be described by the momentary relation

$$U_{wy1} = -i_x R_p - (I_{sz1} + I_m - I_{B1})R_p + k_{u1}(U_{sz1} + U_{os1}) + U_{szR_p}, \tag{27}$$

where: I_{B1} – input bias current of the amplifier W1, I_{sz1} – momentary value of the input noise current of the amplifier W1, I_m – current coming from constructing and circuit elements (i.e. leakage of insulators, cables, printed circuit board), U_{szR_p} – momentary value of thermal noise voltage of the scaling resistor R_p, U_{sz1} – momentary value of input noise voltage of the amplifier W1, U_{os1} – input offset voltage of the amplifier W1, k_{u1} – voltage gain coefficient of the amplifier W1 given by (24).

The voltage U_{wy1} is applied to the second stage (normalizing amplifier) where further disturbances (noise, offset) can occur. In the second stage the polarization of the signal is changed and, after amplification, the output voltage of the normalizing amplifier W2 is equal to

$$U_{wy2} = (U_{wy1} + U_{sz2} + U_{os2})k_{u2}, \tag{28}$$

where symbols U_{sz2} and U_{os2} denote momentary values of input noise and offset voltage of the amplifier W2, as presented in Fig.5. In the above expression the input bias current and input noise current of amplifier W2 as well as the thermal noise of resistors R_1 and R_2 were not taken into consideration. The influence of input current of amplifier W2 is negligible because the output resistance of the first stage is close to zero. In turn, the resistors mentioned have low value of resistance and therefore the noise voltage from these resistors is very small. In the second block of the converter only disturbing voltage sources have a noticeable influence, i.e. offset U_{os2} and noise voltage U_{sz2} of amplifier W2. Such voltage signals add directly to the output voltage U_{wy1} and, after amplification, they can generate additional fluctuations in the reading block of the picoammeter. For this reason the normalizing amplifier should be built using a low-noise precise operational amplifier, e.g. OP177. For such selection of W2 the influence of the offset voltage and noise voltage can be neglected and Eq. (28) can be simplified to the form

$$U_{wy2} = U_{wy1} k_{u2}. \tag{29}$$

As can be concluded from previous considerations, the most important influence on the conversion accuracy have sources of parasitic currents and voltage drops occurring in the first block, therefore the next section will be devoted to the analysis of these effects on the i/u conversion.

In the Eq. (27) describing the voltage drop at the output of the active i/u converter three basic components can be selected: a basic term $i_x R_p$ coming from the measured current and two groups of voltage drops generated by the current and voltage disturbances, respectively. The first group contains disturbing currents flowing to the active input terminal of the converter: input bias current I_{B1} and input noise current I_{sz1} of the electrometric amplifier W1, equivalent leakage currents of insulators and cables, piezoelectric and triboelectric effects denoted as I_m, etc. These currents can be treated as active current sources connected to the measuring circuit in the way presented in Fig. 5. The second group covers parasitic voltage signals: input voltage disturbances of the amplifier (U_{sz1} and U_{os1}) and thermal noise voltage U_{szR_p} of the scaling resistor R_p.

Now we want to analyze phenomena generating parasitic disturbing currents in the active i/u converter.

6.2. Input parasitic currents in the active i/u converter

Input bias current of the amplifier W1

Similarly to the passive i/u converter, the input current of amplifier W1 can be expressed by (19) but the component depending on the input voltage is close to 0. Therefore the most disturbing influence on the accuracy has the input bias current of the amplifier W1. In the best monolithic operational amplifiers the value of this current is not lower than 10÷20 fA and in special selected items even 5÷10 fA. In hybrid electrometric amplifiers of the Keithley Company the input bias current can reach even 1÷2 fA [4]. There exists one more possibility to minimize this current several times using a compensation circuit introduced in [5, 9], nevertheless the efficiency of such a process is not high because the input bias current features a thermal and time drift which makes the compensation imprecise.

Input noise current of amplifier W1

The input noise current of the amplifier W1 is presented in Fig. 5 as a current source connected to the active input terminal of the i/u converter. This source contains two components: low-frequency noise with power spectral density of 1/f type and white noise with constant power spectral density in the whole bandwidth of the amplifier. Data about both these types of noise are usually published by manufacturers. Noise of 1/f type, dominating at low frequencies, is characterized by the peak-to-peak value of current in the pass-band of 0.1÷10 Hz, which for the best electrometric operational amplifiers, e.g. OPA-128 or AD-549, equals 2.3 fA_{p-p}. White noise is described by the mean-square-value of the power spectral density, which for the best electrometric amplifiers is equal to 0.1 $fA/\sqrt{Hz}$. On the most sensitive ranges of picoammeters, the pass-band of the amplifier equals a few Hz only. Then noise of 1/f type has the biggest impact. In turn, for medium and higher ranges, where the pass-band is extending to a few kHz, the resulting noise current is even equal to 20 fA_{p-p}. A detailed explanation of this effect can be found in [5].

As can be deduced from the above considerations, on the 1 pA range the influence of noise current is significant and can result in a fluctuation of picoammeters's readings. In contemporarily manufactured picoammeters this effect is minimized by using statistical methods based on series of measurements [4, 9] with a different number of samples (so-called 'digital filtering').

Leakage currents of printed board insulation

Every electronic circuit, also the circuit of the i/u active converter, is mounted most often on a printed board. The monolithic operational amplifier used as the i/u converter is located in a typical package where all input terminals of the operational amplifier and the power terminals are very close to each other. After installation of the operational amplifier on the printed epoxy board, the small distance between terminals causes a high surface leakage current and cross-leakage current flowing between the $U_c = \pm 15$ V and the input active terminal of the operational amplifier, as shown in Fig. 6.

Sometimes the board's leakage current can be bigger then the measured current. Even for a printed circuit board made of epoxy laminate, the resistance between contact pin 2 (active input of operational amplifier) and contact pins 4 and 7 (voltage supply $\pm U_c$) is not higher than 10^{14} Ω. The supply voltage, e.g. ±15 V, can generate currents flowing to the active input (contact pin 2) of up to ±1.5 pA. The insulation resistance between contact pins 2 and 4, 7 depends also on surface contamination (smudge of dirt, flux, oils, salts, fingerprints, condensation of humidity) and can unpredictably change the value of the leakage current. Fortunately, the leakage current of a board may be reduced to a negligible level by use of shielding of the active input (contact pin 2) [9]. A protective shield is connected to the guard mass at null potential. For this reason the active input has to be soldered not directly to the printed board but to the separated connector located on an additional Teflon (PTFE) insulator pressed into a metallized hole, as shown in Fig. 6b. The metallized hole is surrounded by a protective conductor path with null potential. Then all leakage insulation currents generated by the supply voltage $\pm U_c$ flow to metallized shield, not to the active input. The scaling resistor R_p also cannot be mounted directly on the board but it has to be located on a Teflon insulator, too, just like the operational amplifier.

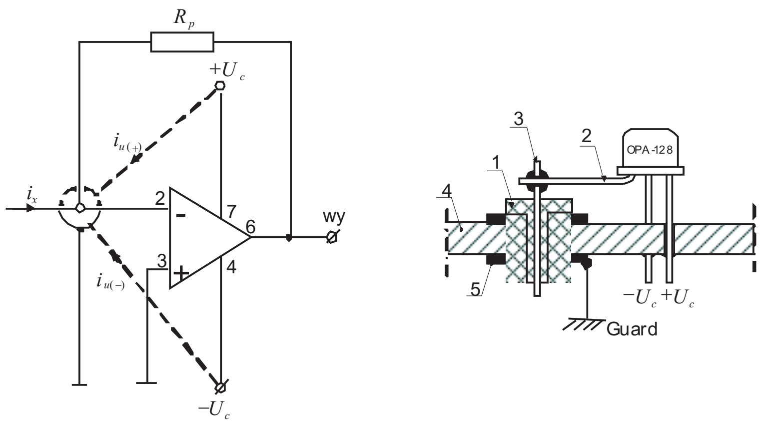

Fig. 6. Leakage currents in the insulating material of the epoxy printed circuit board: a) leakage tracks between input and supply connections, b) method of elimination of current-generating phenomena: 1 – Teflon (PTFE) insulator, 2 – active connector of operational amplifier, 3 – contact pin, 4 – epoxy board, 5 – protective conductor path.

Disturbing current generated by the piezoelectric effect

The use of insulators pressed in the epoxy board (Fig. 6) implies active generation of disturbing currents in the input, coming from the piezoelectric effect. Most often the material used in design of insulators is Teflon which is hardly susceptible to the piezoelectric phenomenon. During the process of pressing the insulator into the board, the induced charge in the active pin can evoke a float of current of even 1 fA ÷ 1 pA to the input of the picoammeter. For this reason the mentioned insulator should not to be tightly pressed into the hole. Moreover, it is necessary to avoid stretching of wires on the printed board.

6.3. Parasitic voltage disturbances in the active i/u converter

As mentioned earlier, in the circuit presented in Fig. 5 three voltage sources of parasitic signals are indicated: input offset and noise voltage of amplifier W1 and thermal noise voltage of the high-ohm scaling resistor R_p. They can disturb the output voltage U_{wy1} of the first stage in a significant way, and, consequently disturb the i/u conversion process.

The amplifier W1 used in the active i/u converter has to be an electrometric operational amplifier. Such an amplifier (monolithic or hybrid) features ultra-low input current and high input resistance. However, its noise and offset parameters are not very good. This is the result of using FET transistors in the input differential amplifier.

Input offset voltage of the amplifier W1

The input offset voltage of the best monolithic electrometric amplifiers usually equals ± 0.5 mV and features considerable thermal drift, typically ± (5÷20) μV/°C. In hybrid amplifiers the offset voltage as well as thermal drift of the offset voltage are even greater, for example in a high-voltage hybrid amplifier from Keithley (not offered for sale) the input offset voltage equals 1÷5 mV and its thermal drift is equal to 25÷30 μV/°C [4]. In any operational amplifier the input offset voltage must be compensated using an external circuit. Then, after initial compensation, the role of the offset voltage plays only its thermal and time drift. The most meaningful is the thermal drift, which e.g. in an ambient temperature interval of about ±10°C the offset voltage is already significant, at the level of 0.1 mV. The time drift manifests itself only in longtime measurement, e.g. longtime monitoring of small current, but in the majority of measurements it can be neglected. Moreover, the input offset voltage is transformed to the output of the amplifier with an amplification factor $k_{u1} = 1 + \frac{R_p}{R_s}$, greater than 1, see (24). Because of the low internal resistance of the source of measured current $i_x (R_s << R_p)$ this factor can be large, even $10^3 \div 10^4$, what implies a considerable voltage drift of zero level in the output voltage of the i/u active converter.

Input noise voltage of the amplifier W1

The internal noise voltage of the amplifier W1 can be expressed as an input voltage source U_{sz1} in the way presented in Fig. 5. Similarly to the earlier mentioned noise current of the amplifier, its voltage source has two components: type 1/f low-frequency noise and white noise with constant power spectral density [10]. Type 1/f noise is defined by manufacturers in the band 0.1÷10 Hz as a maximal peak-to-peak voltage, for example for the OPA-128 amplifier it is about 4 μV_{p-p}. In turn, white noise for the mentioned monolithic amplifiers is equal to 30 nV/$\sqrt{Hz}$ for the pass-band of 10 Hz ÷ 1 kHz. The resultant noise voltage for the selected pass-band is defined as the geometric sum of root-mean-squares of white noise and 1/f-noise voltages. On the most sensitive ranges (1, 10 and 100 pA) the pass-band is not very wide, therefore the effective noise voltage equals 2.5÷4.4 μV_{p-p}. By comparison with monolithic operational amplifiers, the effective noise voltage in a Keithley hybrid amplifier is higher and it can reach 9 μV_{p-p}.

The noise voltage is amplified with the same factor as the previously considered offset voltage and by improper configuration of the measurement circuit it can be really high. The amplified noise voltage at the output of the i/u converter adds to the voltage coming from the measured current i_x causing momentary fluctuations of this voltage.

Thermal noise of the high-ohm scaling resistor R_p

Thermal noise of the scaling resistor R_p with a high value of resistance is significant and its influence cannot be neglected. The root-mean-square value of the noise voltage within a specific pass-band can be calculated from the same formula as for white noise [10]

$$U_{szRp} = \sqrt{4kTR_p\Delta f}, \tag{30}$$

where: $k = 1.38 \cdot 10^{-23}[J/K]$ – Boltzmann constant, T – absolute temperature, Δf – pass-band of the picoammeter, R_p – resistance of the scaling resistor.

On the 1 pA range the pass-band of the picoammeter does not exceed 3 Hz. Then the peak-to-peak value of the thermal noise at ambient temperature equals 423 μV_{p-p} for resistance $R_p = 10^{11}$ Ω [11]. On the 10 pA range the thermal noise U_{szRp} equals 244 μV_{p-p} for $\Delta f = 10$ Hz and $R_p = 10^{10}$ Ω. Thermal noise has the same value on the next ranges, up to 1 nA, because the decreasing value of the scaling resistor is compensated by an increase of the pass-band width. The thermal noise voltage becomes smaller only on ranges higher than 1 nA.

In an active i/u converter this noise voltage appears directly, without amplification, at the output of the amplifier and adds to the output voltage of the converter, causing fluctuations of this voltage. For this reason the minimal value of the output voltage U_{wy1} of the converter should not be lower than 100÷200 mV. There exists a possibility of reducing the noise voltage but it is realized most often by narrowing of the pass-band of the instrument. However, exaggerated narrowing of the pass-band is not recommended in practice because it limits the possibility of registration of fast changes of current which is required in many applications.

6.4. Sources of disturbances appearing in cables

Triboelectric effect

Apart from internal sources of disturbances appearing in the active i/u converter, considered earlier – generated by the amplifier, scaling resistor, construction elements

– there occur also disturbances generated by cables leading the measured signal and disturbances induced in the high-resistance input circuit by external electromagnetic fields. In concentric cables leading the measured signal to the input of the picoammeter, as the result of flection the so-called "triboelectric effect" occurs. This phenomenon is created by frictional effects between the conductor in a low noise cable and the insulator between the inner conductor and the outer shield. As a result of the friction, unbalanced charges in the input circuit (i/u converter) occur. The value of the current in special electrometric cables is greatly reduced to a few fA, although in standard cables it can reach 10 pA ÷ 10 nA. The effect can be minimized by introducing the inner insulator or polyethylene coated with graphite underneath the outer shield. During measurement, cables should be protected from vibrations and flexing.

Electromagnetic disturbances

A different kind of voltage disturbances are voltages induced in the high-resistance input circuit and leading cables by external electromagnetic fields, especially by low-frequency fields and the power network. These disturbances are amplified in amplifier W1 and add to the output voltage of the i/u converter. Such parasitic signals can be reduced by magnetic shielding of the input circuit of the picoammeter, the connecting cables and source of measured current. The impact of such disturbances can be additionally minimized by digital filtering of the signal and statistical processing of data based on a series of measurements.

6.5. Availability and metrological parameters of high-ohm scaling resistor

A key point in the design of picoammeters with extremely low 1÷10 pA ranges is the availability of small-dimension high-resistance precise resistors with very high value of resistance, even $10^{10} \div 10^{11}$ Ω. The metrological parameters of a high-ohm resistor – its accuracy, temperature resistance coefficient (TCR) and voltage resistance coefficient (VCR) – have a dominant impact on the accuracy of the i/u conversion.

So far resistors with high resistance value, and at the same time with small TCR and VCR were hardly available and very expensive. Moreover, for values greater than $10^{10} \div 10^{11}$ Ω their accuracy was dramatically worse [2]. However in the last decade the situation became much better, because precise high-ohm resistors of new generation, so-called oxide MOX resistors with satisfactory metrological parameters, were offered for sale. Such resistors up to the value of $10^{11} \div 10^{12}$ Ω feature an accuracy of 1÷2%, which in special items can reach even 0.2÷0.5%. Their voltage resistance coefficient VCR is not higher than a few ppm/V and the temperature resistance coefficient TCR is also small – up to 50÷200 ppm/°C. For the highest reachable values $10^{11} \div 10^{12}$ Ω their accuracy is worse and equals 2÷5%, moreover TCR and VCR increase noticeably.

It is worth to mention that formerly applied composite resistors have been essentially improved – their metrological parameters are really better – but they constantly are not as good as MOX resistors. This concerns in particular the voltage resistance coefficient VCR [2]. Practically, composite resistors can have higher values of resistance than MOX resistors, up to 10^{14} Ω, however such high values are not needed for the construction of picoammeters because the most sensitive ranges of such device (1÷10 pA) are limited first of all by parameters of operational amplifier, not the high-ohm resistor. This is the cause that existing oxide MOX resistors fulfil all requirements which active i/u converters in the most sensitive picoammeters need.

7. ASSESSMENT OF PICOAMMETER'S UNCERTAINTY

As shown in section 6, the main source of errors occurring in sensitive picoammeters with analog active i/u conversion is the converter which processes current into voltage. The detailed analysis presented in the previous section shows that using appropriate techniques in the design of the instrument and in the structure of the measurement circuit (e.g. shielding) can eliminate parasitic leakage currents and disturbing electromagnetic fields. Moreover, disturbances occurring in the normalizing amplifier can also be neglected in further considerations, if the amplifier W2 would be chosen as a precise low-noise operational amplifier.

To evaluate the uncertainty of the measured current value, we will use an expression for i_x obtained from combination of (27) and (29) in the following way:

$$i_x = \frac{U_{wy2}}{|k_{u2}| R_p} + I_{B1} - I_{sz1} + \frac{k_{u1}}{R_p}(U_{sz1} + U_{os1}) + \frac{U_{szR_p}}{R_p}. \tag{31}$$

For such a form of the measurement function, the combined standard uncertainty of the measured value of current i_x can be given by the general expression [11] as

$$u_c(i_x) = \sqrt{\sum_{j=1}^{9} \left(\frac{\partial i_x}{\partial x_j}\right) u^2(x_j)}, \tag{32}$$

where: $a_j = \left(\frac{\partial i_x}{\partial x_j}\right)$ – sensitivity coefficient describing the influence of j^{th} variable on the measured current i_x, $u(x_j)$ – standard uncertainty of variable x_j.

After calculations we get a detailed form of expression (32) for different a_j as follows:

$$a_1 = \left(\frac{\partial i_x}{\partial k_{u1}}\right) = \frac{U_{sz1} + U_{os1}}{R_p}$$ – sensitivity coefficient describing the influence of the amplification factor k_{u1} on the measured current i_x,

$$a_2 = \left(\frac{\partial i_x}{\partial k_{u2}}\right) = \frac{-U_{wy2}}{k_{u2}^2 R_p}$$ – sensitivity coefficient describing the influence of the amplification factor k_{u2} on the measured current i_x,

$$a_3 = \left(\frac{\partial i_x}{\partial R_p}\right) = \frac{-U_{wy2}}{k_{u2} R_p^2} - \frac{U_{szR_p} + k_{u1}(U_{sz1} + U_{os1})}{R_p^2}$$ – sensitivity coefficient describing the influence of the scaling resistor R_p on the measured current i_x,

$$a_4 = \left(\frac{\partial i_x}{\partial U_{wy2}}\right) = \frac{1}{k_{u2} R_p}$$ – sensitivity coefficient describing the influence of the voltage U_{wy2} on the measured current i_x,

$$a_5 = \left(\frac{\partial i_x}{\partial I_{B1}}\right) = 1$$ – sensitivity coefficient describing the influence of input bias current I_{B1} on the measured current i_x,

$$a_6 = \left(\frac{\partial i_x}{\partial U_{szR_p}}\right) = \frac{1}{R_p}$$ – sensitivity coefficient describing the influence of noise voltage occurring in scaling resistor on the measured current i_x,

$$a_7 = \left(\frac{\partial i_x}{\partial U_{sz1}}\right) = \frac{k_{u1}}{R_p}$$ – sensitivity coefficient describing the influence of input voltage noise of the amplifier W1 on the measured current i_x,

$$a_8 = \left(\frac{\partial i_x}{\partial U_{os1}}\right) = \frac{k_{u1}}{R_p}$$ – sensitivity coefficient describing the influence of input offset voltage U_{os1} of amplifier W1on the measured current i_x,

$$a_9 = \left(\frac{\partial i_x}{\partial I_{sz1}}\right) = -1$$ – sensitivity coefficient describing the influence of input current noise of amplifier W1 on the measured current i_x.

7.1. Uncertainty caused by systematic errors

The combined standard uncertainty of measurement of the current i_x given by Eq. (32) depends on two types of variables with different distributions. In this section we want to discuss uncertainties of physical phenomena which generate systematic errors in the value of the measured current.

From considerations presented in section 6 it can be concluded that this group covers five components with sensitivity coefficients $a_1 - a_5$. The uncertainty resulting from such effects (it is the so-called uncertainty of type B) has been determined as for rectangular distribution, i.e.

$$u(x) = \frac{\Delta x}{\sqrt{3}}, \tag{33}$$

where Δx is the maximal error of variable x, which has been evaluated in section 6. It means that standard uncertainties of these influent variables are equal to:

$u(k_{u1}) = \dfrac{\Delta k_{u1}}{\sqrt{3}}$ – standard uncertainty of amplification factor k_{u1},

$u(k_{u2}) = \dfrac{\Delta k_{u2}}{\sqrt{3}} = \dfrac{(kl_{R1} + kl_{R2})k_{u2}}{\sqrt{3}}$ – standard uncertainty of amplification factor k_{u2} caused by inaccuracy of standard resistors R_1 and R_2 included in the negative feedback-loop of amplifier W2,

$u(I_{B1}) = \dfrac{\Delta I_{B1}}{\sqrt{3}}$ – standard uncertainty of the input bias current,

$u(U_{wy_2}) = \dfrac{\Delta U_{wy2}}{\sqrt{3}}$ – standard uncertainty of the readout of voltage U_{wy2},

$u(R_p)$ – standard uncertainty of scaling resistor R_p.

Standard uncertainty of amplification factor k_{u1}

The standard uncertainty of the amplification factor k_{u1} can be determined from the maximal interval of its value. From considerations presented in section 5.2 we know that $\Delta k_{u1} = 1$, because $1 \leq k_{u1} \leq 2$. It means that

$$u(k_{u1}) = \frac{1}{\sqrt{3}}. \tag{34}$$

Standard uncertainty of amplification factor k_{u2}

The standard uncertainty of the amplification factor k_{u2} depends on accuracy and values of the resistors R_1 and R_2. For calculations, a value $k_{u2} = 10$ realized by resistors $R_1 = 1$ kΩ ±0.01%, and $R_2 = 10$ kΩ ± 0.01% has been chosen. Such small values of resistance can be obtained with very good tolerance, therefore $kl_{R1} = kl_{R2} = 0.01$. It means that

$$u(k_{u2}) = \frac{2 \cdot 10^{-3}}{\sqrt{3}}. \tag{35}$$

Standard uncertainty of input bias current I_{B1}

The input bias current is constant on each range of the picoammeter and for the best monolithic operational amplifiers it reaches a level 10 fA. Then the standard uncertainty of this current is equal to

$$u(I_{B1}) = \frac{10^{-14}}{\sqrt{3}} \text{ A}. \tag{36}$$

Standard uncertainty of voltage U_{wy2}

The standard uncertainty of the output voltage can be characterized by standard uncertainty of the voltage obtained from digital voltmeter readings, given by the above expression

$$u(U_{wy2}) = \frac{\Delta U_{wy2}}{\sqrt{3}}, \tag{37}$$

where: $\Delta U_{wy2} = aU_{wy2} + bU_z$ – the maximal error of digital voltmeter readings (multiplicative error aU_{wy2} – fraction of reading, additive error bU_z – fraction of range).

For a digital voltmeter with $a = 0.005\%$ and $b = 0.001\%$, the voltage U_{wy2} at the end of the range, i.e. $U_{wy2} = U_z = 1$ V can be measured with standard uncertainty

$$u(U_{wy2}) = \frac{60}{\sqrt{3}} \mu\text{V}. \tag{38}$$

Standard uncertainty of scaling resistor R_p

The standard uncertainty of the scaling resistor R_p used in the active i/u converter can be found with the use of the following expression

$$u(R_p) = \frac{1}{\sqrt{3}} \sqrt{(kl_{R_p} R_p)^2 + (TCR \cdot \Delta T \cdot R_p)^2 + (VCR \cdot \Delta U \cdot R_p)^2}, \tag{39}$$

where: kl_{R_p} – accuracy of the scaling resistor R_p determined by the manufacturer in reference conditions (usually $T = 23°\text{C}$ and $U = 100$ V), TCR, VCR – temperature and voltage resistance coefficients of the resistor R_p.

High value scaling resistors, for example $R_p = 10^{11}\ \Omega$ or $R_p = 10^{10}\ \Omega$, required for the most sensitive ranges of picoammeter, feature a relatively low accuracy of 0.5÷2% as well as high temperature (TCR = 100 ppm/°C) and voltage (VCR = 5 ppm/V) resistance coefficients, respectively. For smaller values of resistance the parameters can be better i.e. for $R_p = 10^9\ \Omega$ TCR can reach 50 ppm/°C and VCR = 2 ppm/V [2]. When small variations of nominal voltage and ambient temperature are considered, however, the uncertainty will be proportionally small regarding the systematic error determined by the accuracy of the resistor R_p. Using (39) the standard uncertainty of selected values of R_p can be calculated for admissible changes of temperature and voltage ($\Delta T = \pm 10°\text{C}$, $\Delta U = 100$ V) as follows

$$
\begin{aligned}
u(R_p) &= \frac{1.021 \cdot 10^9}{\sqrt{3}}\Omega \quad \text{for} \quad R_p = 10^{11}\Omega \pm 1\% \quad \text{(range 1 pA)},\\
u(R_p) &= \frac{5.41 \cdot 10^7}{\sqrt{3}}\Omega \quad \text{for} \quad R_p = 10^{10}\Omega \pm 0.5\% \quad \text{(range 10 pA)},\\
u(R_p) &= \frac{5.10 \cdot 10^6}{\sqrt{3}}\Omega \quad \text{for} \quad R_p = 10^{9}\Omega \pm 0.5\% \quad \text{(range 100 pA)}.
\end{aligned} \tag{40}
$$

7.2. Uncertainty caused by random errors

The combined standard uncertainty of the measurement of current i_x given by Eq. (32) depends also on uncertainties of physical phenomena which generate random errors in the value of the measured current. From considerations presented in section 6 it can be concluded that this group covers four factors with sensitivity coefficients $a_6 - a_9$. The uncertainty resulting from random effects (it is the so-called uncertainty of type A) has be determined as for Gaussian distribution, assuming coverage probability $p = 0.99$, in the following manner [12]

$$u(x) = \frac{\Delta x_{\max}}{3}, \tag{41}$$

where $\Delta x_{\max}$ is the maximal error, which has been evaluated in section 6. If noise is taken into consideration, then $\Delta x_{\max} = \dfrac{\Delta x_{p-p}}{2}$.

Consequently, standard uncertainties of influent variables are equal to

$u(U_{szR_p}) = \dfrac{\Delta U_{szR_p p-p}}{6}$ – standard uncertainty of thermal noise of the scaling resistor R_p,

$u(U_{sz1}) = \dfrac{\Delta U_{sz1p-p}}{6}$ – standard uncertainty of voltage noise of the amplifier W1,

$u(I_{sz1}) = \dfrac{\Delta I_{sz1p-p}}{6}$ – standard uncertainty of current noise of the amplifier W1,

$u(U_{os1}) = \dfrac{\Delta U_{os1}}{3}$ – standard uncertainty of offset voltage of the amplifier W1.

As shown in section 6.3, all noisy phenomena depend on the pass-band of the selected range of the instrument. For this reason in further considerations the standard uncertainties are calculated for the following pass-band: $\Delta f = 3$ Hz on the 1 pA range, $\Delta f = 10$ Hz on the 10 pA range and $\Delta f = 100$ Hz on the 100 pA range.

Standard uncertainty of thermal noise of the scaling resistor R_p

The standard uncertainty of thermal noise depends not only on the pass-band of the range but on the value of the scaling resistor as well. If values of thermal noise for selected ranges of the picoammeter are taken from section 6.3, then the standard uncertainty of U_{szR_p} equals

$$
\begin{aligned}
u(U_{szR_p}) &= \frac{423}{6}\mu\text{V} \quad \text{for} \quad R_p = 10^{11}\Omega \text{ and } \Delta f = 3 \text{ Hz} \quad \text{(range 1 pA)},\\
u(U_{szR_p}) &= \frac{244}{6}\mu\text{V} \quad \text{for} \quad R_p = 10^{10}\Omega \text{ and } \Delta f = 10 \text{ Hz} \quad \text{(range 10 pA)},\\
u(U_{szR_p}) &= \frac{244}{6}\mu\text{V} \quad \text{for} \quad R_p = 10^{9}\Omega \text{ and } \Delta f = 100 \text{ Hz} \quad \text{(range 100 pA)}.
\end{aligned} \tag{42}
$$

Standard uncertainty of voltage noise of the amplifier W1

The standard uncertainty of U_{sz1} can be evaluated using ΔU_{sz1p-p} obtained in section 6.3.

$$u(U_{sz1}) = \frac{2.5}{6}\mu\text{V} \quad \text{(range 1 pA)},$$
$$u(U_{sz1}) = \frac{4}{6}\mu\text{V} \quad \text{(range 10 pA)}, \tag{43}$$
$$u(U_{sz1}) = \frac{4.4}{6}\mu\text{V} \quad \text{(range 100 pA)}.$$

Standard uncertainty of current noise of the amplifier W1

The standard uncertainty of I_{sz1} can be evaluated using ΔI_{sz1p-p} obtained in section 6.2.

$$u(I_{sz1}) = \frac{1.7}{6}\text{fA} \quad \text{(range 1 pA)},$$
$$u(I_{sz1}) = \frac{2.3}{6}\text{fA} \quad \text{(range 10 pA)}, \tag{44}$$
$$u(I_{sz1}) = \frac{6.43}{6}\text{fA} \quad \text{(range 100 pA)}.$$

Standard uncertainty of offset voltage of the amplifier W1

During the operation of the picoammeter, the internal temperature in the instrument changes in an unexpected way, therefore it can be assumed that these variations are random processes. It implies a random character of changes of the offset voltage drift which results from fluctuations of temperature. For calculations, the assumption is made that maximal changes of temperature do not exceed $\Delta T = \pm 10°\text{C}$ and that the drift of offset voltage of the operational amplifier is at the level of $\pm 5\mu\text{V}/°\text{C}$. For such selection of conditions the maximal changes of U_{os1} are limited to the value 100 μV. In such case the standard uncertainty of the offset voltage can be expressed on any range as follows

$$u(U_{os1}) = \frac{100}{3}\mu\text{V}. \tag{45}$$

7.3. Impact of different factors on standard uncertainty of picoammeters on the lowest ranges

The combined standard uncertainty $u_c(i_x)$ of the measured current given by Eq. (32) depends not only on partial uncertainties of influencing input quantities but on sensitivity coefficients a_j, too. For better illustration of the impact of different components on the combined standard uncertainty of the picoammeter, appropriate factors will be presented in a systematic way in Table 1.

Calculations have been made for three most sensitive ranges (1 pA, 10 pA, 100 pA) of the best contemporarily manufactured picoammeters.

Table 1. Impact of different factors on the accuracy of picoammeters on different ranges.

Range	1 pA $R_p = 10^{11}\ \Omega \pm 1\%$	10 pA $R_p = 10^{10}\ \Omega \pm 0.5\%$	100 pA $R_p = 10^{9}\ \Omega \pm 0.5\%$
$a_1u(k_{u1})$	0.592 fA	6.004 fA	60.275 fA
$a_2u(k_{u2})$	−0.116 fA	−1.16 fA	−11.6 fA
$a_3u(R_p)$	−5.93 fA	−32.4 fA	−295.8 fA
$a_4u(U_{wy2})$	0.035 fA	0.35 fA	3.5 fA
$a_5u(I_{B1})$	5.774 fA	5.774 fA	5.774 fA
$a_6u(U_{szRp})$	0.705 fA	4.067 fA	40.67 fA
$a_7u(U_{sz1})$	0.004 fA	0.067 fA	0.732 fA
$a_8u(U_{os1})$	0.333 fA	3.33 fA	33.3 fA
$a_9u(I_{sz1})$	0.283 fA	0.383 fA	1.07 fA
combined standard uncertainty $u_c(i_x)$ of measured current	8.34 fA	33.89 fA	306.7 fA

7.4. Expanded uncertainty of measured current at the lowest ranges of picoammeter

The accuracy of a picoammeter on any range, stated by the manufacturer, is defined as so-called expanded uncertainty [12]

$$U(i_x) = k_p u_c(i_x), \tag{46}$$

where: $U(i_x)$ – expanded uncertainty of measured current determined on a selected picoammeter range, k_p – coverage factor, $u_c(i_x)$ – combined standard uncertainty of measured current on the range.

Choosing the proper coverage factor is the most difficult problem when evaluating the expanded uncertainty. This factor depends on the assumed coverage probability and accurate determination of convolution of distributions of input quantities having a real impact on the resulting measurement of current.

In the considered picoammeter with an active i/u converter on the 1 pA range the most influent on the accuracy are two systematic errors generated by the scaling resistor R_p and the input bias current of the operational amplifier W1. The errors have approximately similar rectangular distributions, therefore the resulting distribution of current uncertainty can be considered to be convergent to triangular (Simpson) distribution characterized by the coverage factor $k_p = \sqrt{6}p$, where p is the assumed coverage probability.

However, on the 1 pA range a comparable impact on the accuracy have additionally three components with Gaussian distribution (voltage noise and offset voltage drift in the amplifier W1 and thermal noise of the scaling resistor) and one component with rectangular distribution (k_{u1} coefficient). In such a case, due to the Lindeberg-Levy theorem, the distribution of uncertainty will be more complex and it will be more safe then to assume that the distribution is normal rather than triangular and to assume a coverage factor $k_p = 3$ for $p = 0.99$.

On the 10 pA and 100 pA ranges the dominant impact has only one component, namely the systematic error caused by uncertainty of the scaling resistor R_p. Nevertheless, on these ranges other errors are considerably high, i.e. components with normal (U_{szR_p},U_{os1}) and rectangular distribution (k_{u1}) and on the 10 pA range the systematic error additionally coming from I_{B1}. In such situation it is justified to take a coverage factor as for Gaussian distribution, i.e. $k_p = 3$ for $p = 0.99$. It means that the expanded relative uncertainty on the lowest ranges of the picoammeter is equal to

$$\begin{aligned}
\frac{U(i_x)}{i_x} &= \frac{3u_c(i_x)}{i_x} = \frac{3 \cdot 8.43 \cdot 10^{-15}}{10^{-12}} = 2.5\% \quad \text{(for } i_x = I_z \text{ on the 1 pA range)},\\
\frac{U(i_x)}{i_x} &= \frac{3u_c(i_x)}{i_x} = \frac{3 \cdot 33.89 \cdot 10^{-15}}{10^{-11}} = 1\% \quad \text{(for } i_x = I_z \text{ on the 10 pA range)},\\
\frac{U(i_x)}{i_x} &= \frac{3u_c(i_x)}{i_x} = \frac{3 \cdot 306.7 \cdot 10^{-15}}{10^{-10}} = 0.9\% \quad \text{(for } i_x = I_z \text{ on the 100 pA range)}.
\end{aligned} \tag{47}$$

8. SUMMARY AND CONCLUDING REMARKS

According to expectations, on the most sensitive (lowest) ranges of the picoammeter, the most significant is the influence of errors coming from the scaling resistor R_p and the input bias current of the electrometric operational amplifier W1 which an active i/u converter is built on. These two components limit the most sensitive ranges of contemporary picoammeters to the level of 10÷20 pA with an achieved accuracy of about 1÷2%. An attempt to further lower the ranges implies deterioration of picoammeter's accuracy to a value of 2.5÷5%. On ranges higher than 100 pA the input bias current of contemporarily manufactured electrometric operational amplifiers is already small enough to keep the impact of the phenomenon on the measurement error at a negligible level.

For picoammeters with wide bandwidth, a significant impact on instrument's accuracy has the noise generated by the scaling resistor R_p. The influence of the thermal noise of the resistor R_p is almost constant for the few lowest ranges. However the impact of this noise decreases for ranges higher than 1 nA and then it can be neglected. If the bandwidth would be the same on all ranges, then the impact of such noise would be essential only on the most sensitive ranges.

The influence of temperature drift of the input offset voltage occurs in a different manner. Its effect is the same on all ranges of the device. The time drift of the input offset voltage, which may be noticeable in a long measurement time, has a similar character. The limitation of the influence of these drifts requires to maintain a constant ambient temperature and keeping a short measurement time.

Considerations presented in the paper should introduce users and designers of electrometric equipment to the field of measurements of extremely low DC and low-frequency current. In many situations it should present the users of such special equipment the danger of improper selection of measurement conditions. Moreover this work should make it possible for the users to distinguish disturbances coming from different sources and correctly evaluate the accuracy of obtained results.

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