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CHARACTERIZATION OF DEAD WEIGHT TESTERS AND COMPUTATION OF ASSOCIATED UNCERTAINTIES: A CASE STUDY OF CONTEMPORARY TECHNIQUES

Estimation of uncertainty associated with pressure, measured by a dead weight tester is a key issue of recent times in pressure metrology. Though two different techniques have been used in the past yet there has been little uniformity in the way in which the measurement uncertainties of dead weight piston testers are evaluated and expressed. One of the techniques described in EAL document follows the universal approach based on ISO Guide and the relatively recent approach described in the National Accreditation Board of Laboratories (NABL-141) document presents a method for evaluating uncertainty in pressure measurement using dead weight testers through statistical analysis and error evaluated through linear curve fitting. This method is though in line with ISO Guide on the expression of uncertainty in measurement but differs from the traditional uncertainty measurement computations. The present paper describes a comparative study of these two different approaches through a case study carried out on an industrial dual range simple type dead weight tester. The paper also highlights some of the facts: how the new estimates affect pressure measurement and their commercial implementation and the benefits over traditional estimates.

1. INTRODUCTION

The high-pressure technology and new applications demand increasingly of metrologists that the best instrumentation should ensure the lowest measurement uncertainty, particularly in the fluid media. Dead weight piston gauges are the primary standards to measure high hydrostatic pressure in fluid media from atmospheric to few GPa. Therefore, the dead weight piston gauge, also known as pressure balance, dead weight tester and piston manometer are regarded as a fundamental pressure-measuring instrument with low measurement uncertainty. The essential feature of the device is a cylinder which is closed at the bottom with appropriate seals and plumbing connections to a pressure generating system and closed at the top with a close-fitted piston floating in the fluid media at the specified reference level. The piston is loaded with known weights and rotated to relative friction and assures concentricity. The pressure is then determined as the ratio of the forces to the effective area of the piston-cylinder assembly.

The dead weight piston gauge is generally characterized by a procedure called cross floating, in which it is hydrostatically balanced against a similar standard of known effective cross-sectional area [1–9].

The calculation of associated uncertainties in pressure measurement using cross-floating of pressure balances is a complex matter involving many influence quantities plus the random uncertainties of the calibration. Various documents are available to guide the users to compute the measurement uncertainties of pressure balances [5–9]. To date there has been little uniformity in the ways in which the measurement uncertainties of a dead weight piston gauge are evaluated and expressed. The Nordtest method [5], used in Denmark describes the calibration of pressure balances and covers i) mass determination of the piston and the weights of the pressure balance and ii) determination of the effective area of the piston-cylinder assembly in the pressure range (4–2000 bar). The DKD R 3-4 gives directives for the calibration of pressure balances within the scope of the German calibration services [6]. The OIML R110 applies to pressure balances equipped with either a simple type or a re-entrant type piston-cylinder assembly with direct loading, and which are used for measuring the gauge pressure in the range from 0.1 MPa to 500 MPa, specifying metrological and technical requirements, testing methods and the format of the test report applicable to pressure balances [7].

The technique described in the EAL document is a good attempt to harmonize the calibration of dead weight testers and for the computation of associated uncertainties and is being followed by all laboratories in Europe [8]. This document has been produced to improve the harmonization in pressure measurement. It provides guidance to national accreditation bodies to set up minimum requirements for the calibration of pressure balances and gives advice to calibration laboratories to establish practical procedures. The document contains a detailed example of the estimation of the uncertainty contribution of a pressure balance when used for the calibration of another measuring instrument. Recently, the National Accreditation Board of Laboratories (NABL), India, has introduced the method of evaluating measurement uncertainty in pressure measurement using a dead weight tester through statistical analysis and error evaluated through linear curve fitting [9]. It is worth mentioning here that NABL is imposing stipulation for NABL accredited calibration laboratories in India to follow their guidelines strictly. Therefore, it is important to compare both methods available for the estimation of measurement uncertainty using a dead weight piston manometer. In this context, an industrial dual range simple type dead weight piston manometer has been studied by evaluating measurement uncertainty associated with pressure measurement using both methods described above. In the present paper I shall discuss briefly the comparative results thus obtained.

2. METHODOLOGY

In the present work, an industrial simple type dead weight piston gauge was studied for the measurement uncertainty associated with pressure measurement. In order to estimate the uncertainty associated with pressure measurement, a comprehensive metrological characterization of a dead weight piston gauge is required. The characterization of a dead weight tester starts with the calibration of dead weights for their assigned mass values with measurement uncertainties and is followed by collection of pressure data, computation of pressure generated by a standard and force generated by the instrument under test, determination of effective area, curve fitting of data and finally computation of measurement uncertainty. The calibration of mass values of dead weights (mostly in the denomination of 5.8 kg, made of stainless steel), was performed against appropriate national standards of mass and balances. Calibration of the industrial dead weight tester (GUT) against a standard dead weight tester (DWT) is performed by the method of cross-floating of two dead weight testers, as shown in Fig. 1. Both the dead weight testers, referred as DWT and GUT are placed on a strong rigid table (stainless steel sheet having a thickness of 15 mm) in the calibration room to isolate vibrations. In a cross floating condition, the two dead weight testers are connected together to a pressure line and brought to a common balance at various pressure points to be calibrated. The balancing operation is identical with that employed on an equal arm weighing balance where the mass of one weight is compared to the other. During cross floating, the piston is rotated at 20–30 rpm with the help of an electric motor to reduce the effect of friction. The DWTs are considered to be in balance when the sink rate of the standard piston is close to the original fall rate of the piston when it is isolated from the GUT. At this position, there is no pressure drop in the connecting line and consequently no movement of the fluid.

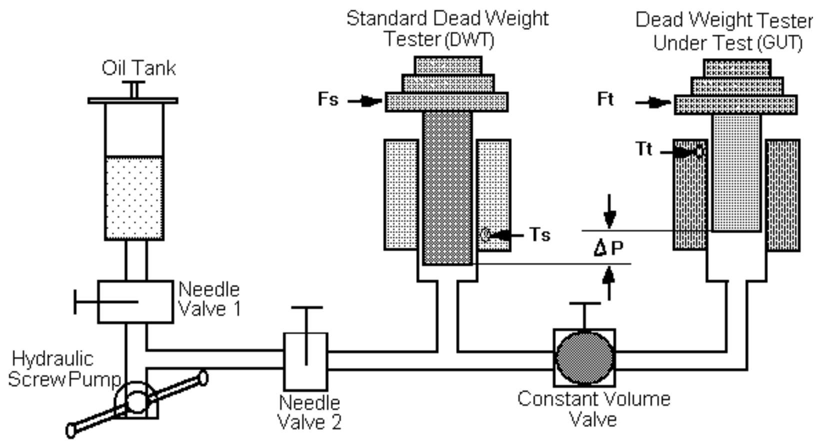

Fig. 1. Calibration set up.

The performance of a dead weight tester is affected by the following factors: elastic distortion of the piston and cylinder; temperature of the piston and cylinder; effect of gravity on the masses; buoyancy effect of the atmosphere upon the masses; hydraulic pressure gradients within the equipment; surface tension effects of the liquids and head correction for the difference of height between two dead weight testers.

Thus, after applying all the corrections mentioned above, the pressure generated/measured by DWT (in Pa) is determined by using the following expression [1–3]:

$$P_{DWT} = \frac{\Sigma_i m_i g_L (1 - \rho_{air}/\rho_{mi}) + \gamma C}{A_0 (1 + bp) \left[\left(\alpha_c + \alpha_p \right) (T - T_r) \right]} \pm \Delta p. \quad (1)$$

The term $(1 - \rho_{air}/\rho_{mi})$ is the air buoyancy correction for weights, γC is the force exerted on the piston by surface tension of the transmitting fluid, $[1 + (\alpha_c + \alpha_p)(T - T_r)]$ is the thermal expansion correction factor, the term $(1 + bp)$ describes the change of effective area with pressure which is the most important correction term. The various terminology used in the equation is defined as follows:

m_i mass of the ith weight combination (in kg) placed on the DWT,

g_L value of local acceleration of gravity (in m/s^2) in the measurement laboratory,

ρ_{air} density of the air (in kg/m^3) at the temperature, barometric pressure and humidity prevailing in the laboratory,

ρ_{mi} density (in kg/m^3) of the material of weights,

γ surface tension (in N/m) of the pressure transmitting fluid used,

C circumference (in m) of the piston where it emerges from the fluid,

A_0 zero pressure effective area (in m^2) of the DWT,

α_c & α_p thermal expansion coefficients (in /°C) of material of cylinder and piston, respectively of the DWT,

T measurement temperature (in °C) of the DWT piston – cylinder assembly,

T_r temperature (in °C) at which A_0 (zero pressure effective area) of DWT is referred,

b pressure distortion coefficient (in per Pa) of the DWT, and

Δp is the head correction (in Pa) in terms of pressure.

The head correction term $\Delta p = [(\rho_f - \rho_{air}) g_L H]$, where H is the difference in height (in m) between the reference levels of the two dead weight testers and (ρ_f) is the density (in kg/m^3) of the pressure transmitting fluid used in the measurements.

3. EVALUATION OF ASSOCIATED UNCERTAINTY USING EAL GUIDELINES

3.1. Uncertainty Associated with Effective Area Measurements

The temperature corrected force F (in N) acting on the Test is calculated using the expression [1–3]:

$$F_{Test} = \frac{\Sigma m_{it} g_L (1 - \rho_{air}/\rho_{mi}) + \gamma C_t}{\left[1 + \left(\alpha_{ct} + \alpha_{pt}\right)(T_t - T_{rt})\right]}, \quad (2)$$

where: m_{it} – mass (in kg) of the ith weight combination placed on the Test, C_t – circumference (in m) of the piston of the Test where it emerges from the fluid, α_{ct} & a_{pt} – thermal expansion coefficients (in /°C) of material of cylinder and piston, respectively of the Test, T_t – measurement temperature (in °C) of the piston – cylinder assembly of the Test, T_{rt} – temperature at which A_{0t} (zero pressure effective area) of the Test is to be calculated.

The effective area A_{eff} (in m^2) of the Test is then calculated from [1–3]:

$$A_{eff} = F_{Test}/P_{DWT}, \quad (3)$$

$$A_{eff} = \frac{\Sigma m_{it} g_L (1 - \rho_{air}/\rho_{mi}) + \gamma C_t}{P_{DWT}\left[1 + \left(\alpha_{ct} + \alpha_{pt}\right)(T_t - T_{rt})\right]}. \quad (4)$$

The data thus obtained was recorded at 14 different pressure points of 7 MPa, 10 MPa, 15 MPa, 20 MPa, 25 MPa, 30 MPa, 35 MPa, 40 MPa, 45 MPa, 50 MPa, 55 MPa, 60 MPa, 65 MPa and 70 MPa and observations were repeated six times at each pressure point. The pressure measured by DWT is least square fitted against the effective area of the Test to determine the value of A_{0t} (zero pressure effective area) and b_t (distortion coefficient) of the Test. The calibration results thus obtained are shown in Table 1 for exemplary four pressure points.

The combined uncertainty associated with effective area measurements is then estimated using:

$$u_c^2\left(A_{eff}\right) = \sum_{i=1}^{n} \left(\frac{\partial A_{eff}}{\partial x_i}\right)^2 u^2(x_i), \quad (5)$$

where $u(x_i)$ represents the i^{th} uncertainty contribution and $\frac{\partial A_{eff}}{\partial x_i}$ is the corresponding sensitivity coefficient derived from partial differentiation of Eq. (4). Further, from the values of A_{0t} and b_t, the pressure generated by the Test is computed using [8]:

$$P_{Test} = \frac{\Sigma m_{it} g_L \left(1 - \rho_{air}/\rho_{mi}\right) + \gamma C_t}{A_{0t} \left(1 + b_t P_N\right) \left[1 + \left(\alpha_{ct} + \alpha_{pt}\right)\left(T_t - T_{rt}\right)\right]}. \tag{6}$$

Table 1. The calibration results obtained.

P_{Test}	P_{DWT}	P_{Mean}	(A_{eff})	$\delta(P_{DWT})$	$\delta(A_{eff})$	$u_A = \delta(D_{WT})/(n)^{1/2}$	$u_A = \delta(A_{eff})/(n)^{1/2}$
(MPa)	(MPa)	(MPa)	m^2	(MPa)	m^2	(MPa)	m^2
7	6.998447	6.998016	9.792332E-06	0.00024381	2.608730E-10	9.95358E-05	1.06501E-10
	6.997836		9.793000E-06				
	6.99808		9.792829E-06				
	6.997842		9.792989E-06				
	6.998075		9.792820E-06				
	6.997813		9.793025E-06				
10	9.99733	9.997060	9.792711E-06	0.00019352	1.245255E-10	7.90049E-05	5.08373E-11
	9.996955		9.792892E-06				
	9.997278		9.792717E-06				
	9.996881		9.792979E-06				
	9.997		9.792987E-06				
	9.996918		9.792923E-06				
....	....	....	....	....	....	....	....
60	59.9908	59.990735	9.791418E-06	0.00036969	1.665704E-11	0.000150925	6.80021E-12
	59.99078		9.791441E-06				
	59.99037		9.791434E-06				
	59.99053		9.791443E-06				
	59.99052		9.791417E-06				
	59.99141		9.791461E-06				
70	69.99106	69.991222	9.791148E-06	0.00042762	2.590623E-11	0.000174574	1.05762E-11
	69.99124		9.791155E-06				
	69.99099		9.791119E-06				
	69.99102		9.791163E-06				
	69.99095		9.791131E-06				
	69.99207		9.791193E-06				
Least squares curve fitting of effective area (A_{eff}) as a function of measured pressure (P_{DWT})							
Zero pressure effective area $A_{0t} = 9.793227 \times 10^{-6}$ m^2					Distortion coefficient $(b_t) = -2.948 \times 10^{-6}$/MPa		
$u_{A2} = u(A_0) = 1.298 \times 10^{-10}$ m^2					$u_{A3} = u(b_t) = 8.35 \times 10^{-8}$/MPa		

The combined uncertainty is then estimated using:

$$u_c^2\left(P_{Test}\right) = \sum_{i=1}^{n} \left(\frac{\partial P_{Test}}{\partial x_i}\right)^2 u^2\left(x_i\right), \tag{7}$$

where $u(x_i)$ represents the i^{th} uncertainty contribution and $\frac{\partial P_{Test}}{\partial x_i}$ is the corresponding sensitivity coefficient derived from partial differentiation of Eq. (6).

3.1.1. Evaluation of Type A Uncertainty

The largest relative standard uncertainty evaluated through Type A method due to repeatability of effective area from Table 1 is $u_A = (Largest\ u_A)/A_{0t}) = 10.88\times10^{-6}A_{0t}$.

3.1.2. Evaluation of Type B Uncertainty

The uncertainty components evaluated through Type B method are the uncertainty associated with mass (m_t) as $u_{B1} = [\{u(m_t)/m_t\}A_{0t}] = 0.72\times10^{-6}A_{0t}$; with acceleration of gravity (g_L) as $u_{B2} = [\{u(g_L)/g_L\}A_{0t}] = 0.11\times10^{-6}A_{0t}$; with air density ($\rho_{air}$) as $u_{B3} = [\{u(\rho_{air})/(\rho_{mt}\}A_{0t}] = 0.46\times10^{-6}A_{0t}$; mass density ($\rho_{mt}$) as $u_{B4} = [\{u(\rho_{mt})(\rho_{air})/(\rho_{mt}^2)\}A_{0t}] = 0.89\times10^{-6}A_{0t}$; with surface tension ($\gamma$) as $u_{B5} = [\{(u(\gamma)C_t)/(mg_L)\}A_{0t}] = 0.003\times10^{-6}A_{0t}$; with circumference ($C_t$) as $u_{B6} = [\{(u(C_t)\gamma)/(m_t g_L)\}A_{0t}] = 0$; with thermal expansion of piston (α_p) as $u_{B7} = [\{u(\alpha_{pt})(T_t - T_{rt})\}A_{0t}] = 0.27\times10^{-6}A_{0t}$; with thermal expansion of cylinder (α_c) as $u_{B8} = [\{u(\alpha_{ct})(T_t - T_r)\}A_{0t}] = 0.27\times10^{-6}A_{0t}$; with temperature difference ($T - T_r$) as $u_{B9} = [\{u(T_t - T_r)(\alpha_{pt} + \alpha_{ct})\}A_{0t}] = 5.3\times10^{-6}A_{0t}$ and with reference pressure measurement (P_{DWT}) as $u_{B10} = [\{u(P_{DWT})/(P_{DWT})\}A_{0t}] = 75\times10^{-6}A_{0t}$. Therefore, the combined relative Type B standard uncertainty, $u_B = 75.2\times10^{-6}A_{0t}$ is then evaluated from the root sum square of all these components.

3.1.3. Combined Relative Standard Uncertainty

The combined relative standard uncertainty $u_c\left(A_{eff}\right) = \sqrt{(u_A)^2 + (u_B)^2} = 76\times10^{-6}A_{0t}$ is then evaluated from the root sum square of all the uncertainty components evaluated through Type A and Type B methods.

3.1.4. Effective Degree of Freedom of $u_c(A_{eff})$

The effective degree of freedom $\upsilon_{eff} = 11904$ is computed using:

$$\upsilon_{eff} = \frac{\left\{u\left(A_{eff}\right)\right\}^4}{\frac{(u_A1)^4}{\upsilon_{A1}} + \frac{(u_{B1})^4}{\upsilon_{B1}} + \frac{(u_{B2})^4}{\upsilon_{B2}} + \ldots\ldots + \frac{(u_{B10})^4}{\upsilon_{B10}}}. \quad (8)$$

3.1.5. Expanded Uncertainty

Using the Student's table, $k = 2$ for a confidence level of approximately 95.45%, the expanded uncertainty is then computed as $U = k\ u_c(A_{eff}) = 152\times10^{-6}A_{0t}$.

3.2. Uncertainty Associated with Pressure Measurements

The pressure measured by the Test (P_{Test}) under calibration conditions and the difference between Standard pressure (P_{DWT}) and Test pressure (P_{Test}) is shown in Table 2 for four exemplary pressure points.

Table 2. Pressure measured by the Test (P_{Test}).

P_{DWT} MPa	P_{Test} MPa	$P_{DWT} - P_{Test}$ MPa	Mean ($P_{DWT} - P_{Test}$) MPa	σ ($P_{DWT} - P_{Test}$) MPa	Type A MPa
6.998447	6.99795	−0.00050	−0.00014	0.000187	0.000076
6.997836	6.99782	−0.00002			
6.99808	6.99794	−0.00014			
6.997842	6.99782	−0.00003			
6.998075	6.99793	−0.00015			
6.997813	6.99781	0.00000			
9.99733	9.99710	−0.00023	−0.00007	0.000127	0.000052
9.996955	9.99691	−0.00005			
9.997278	9.99705	−0.00023			
9.996881	9.99692	0.00004			
9.997	9.99705	0.00005			
9.996918	9.99690	−0.00002			
....	....	....	....	....	....
59.9908	59.99033	−0.00047	−0.00037	9.94E-05	0.000041
59.99078	59.99044	−0.00034			
59.99037	59.99000	−0.00037			
59.99053	59.99020	−0.00033			
59.99052	59.99004	−0.00048			
59.99141	59.99119	−0.00022			
69.99106	69.99064	−0.00042	−0.00040	0.000184	0.000075
69.99124	69.99086	−0.00038			
69.99099	69.99036	−0.00063			
69.99102	69.99071	−0.00031			
69.99095	69.99040	−0.00055			
69.99207	69.99197	−0.00010			

3.2.1. Evaluation of Type A Standard Uncertainty

The largest standard uncertainty through Type A method from Table 1 is u_{A1} $(Largest)$ = 0.000175 MPa. Since the uncertainty components associated with A_{0t}, $u_{A2} = [\{u(A_0)/A_0\}p]$ = 0.000928 MPa and b_t, $u_{A3} = [\{u(b)p\}p]$ = 0.000409 MPa are evaluated through statistical analysis, they are considered as Type A components. The combined standard uncertainty evaluated through Type A method is $u_A = \sqrt{(u_{A1})^2 + (u_{A2})^2 + (u_{A3})^2}$ = 0.00103 MPa.

3.2.2. Evaluation of Type B Standard Uncertainty

The uncertainty components evaluated through Type B method are the uncertainty associated with mass (m) as $u_{B1} = [\{u(m)/m\}p]$ = 0.00005 MPa; with acceleration of gravity (g_L) as $u_{B2} = [\{u(g_L)/g_L\}p]$ = 0.000008 MPa; with air density (ρ_{air}) as $u_{B3} = [\{u(\rho_{air})/(\rho_m\}p]$ = 0.000032 MPa; with mass density (ρ_{air}) as $u_{B4} = [\{u(\rho_m)(\rho_{air})/(\rho_m^2)\}p]$ = 0.000063 MPa; with surface tension (γ) as $u_{B5} = [\{(u(\gamma)C)/(mg_L)\}p]$ = 0.000002 MPa; with circumference (C) as $u_{B6} = [\{(u(C)\gamma)/(mg_L)\}p] = 0$; with thermal expansion of piston (α_p) as $u_{B7} = [\{u(\alpha_p)(T - T_r)\}p]$ = 0.000019 MPa; with thermal expansion of cylinder (α_c) as $u_{B8} = [\{u(\alpha_c)(T - T_r)\}p]$ = 0.000019 MPa; with temperature difference $(T - T_r)$ as $u_{B9} = [\{u(T - T_r)(\alpha_p + \alpha_c)\}p]$ = 0.000368 MPa and nominal pressure $(P_{Nominal})$ as $u_{B10} = [\{u(P_{Nominal})(b)\}p]$ = 0.000001 MPa. The uncertainty contribution due to uncertainty of the DWT as u_{B11} = 0.0053 MPa at 70 MPa at $k = 1$. Therefore, the combined standard uncertainty evaluated through Type B method u_B = 0.0053 MPa which is the root sum square of all the eleven uncertainty components.

3.2.3. Combined Standard Uncertainty

The combined standard uncertainty $u_c\,(P_{Test}) = \sqrt{(u_A)^2 + (u_B)^2}$ = 0.0054 MPa is then evaluated form the root sum square of all the uncertainty components.

3.2.4. Effective Degree of Freedom of $u_c\,(P_{Test})$

The effective degree of freedom υ_{eff} = 64961 is computed in the same way as for Eq. (8).

3.2.5. Expanded Uncertainty

Using the Student's table, $k = 2$ for a confidence level of approximately 95.45%, the expanded uncertainty is then computed as $U = k\,u_c(P_{Test})$ = 0.011 MPa.

Table 3. Uncertainty budget at 70 MPa.

Source of Uncertainty (X_i)	Estimates (x_i)	Limits Δx_i	Probability Distribution/ Type/Divisor	Standard Uncertainty $u(x_i)$	Sensitivity Coefficient	Uncertainty Contribution $u_i(y) \times 10^{-6}$	Uncertainty Contribution $u_i(y)$ (MPa)	Degree of Freedom
Repeatability	70.00	±0.000175 MPa	N / A / 1.0	0.000175 MPa	0.014286 $(1/p)$	2.5	0.000175	5
M (kg)	70.00053	±0.00005 kg	N / B / 1.0	0.00005 kg	0.014286 $(1/M)$	0.72	0.00005	∞
g_{NPL} (m/s^2)	9.7912393	$\pm 1.012 \times 10^{-6}$ m/s^2	N / B / 1.0	1.012×10^{-6} m/s^2	0.10213 $(1/g_{NPL})$	0.11	0.000008	∞
ρ_{air} (kg/m^3)	1.150307	±0.0036 kg/m^3	N / B / 1.0	0.0036 kg/m^3	1.26×10^{-4} $(1/\rho_M)$	0.46	0.000032	∞
ρ_M (kg/m^3)	7920	±80 kg/m^3	R / B / $\sqrt{3}$	46.19 kg/m^3	1.81×10^{-8} $(\rho_{air}/\rho_M{}^2)$	0.89	0.000063	∞
γ (N/m)	0.0309	±0.00309 N/m	R / B / $\sqrt{3}$	0.0018 N/m	1.62×10^{-5} $(C/M.g_{NPL})$	0.003	0.000002	∞
C (m)	7.9197×10^{-3}	$\pm 5.0 \times 10^{-7}$ m	N / B / 1.0	5.55×10^{-7} m	4.51×10^{-5} $(\gamma/M.g_{NPL})$	0	0	∞
A_0 (m^2)	9.793227×10^{-6}	$\pm 1.298 \times 10^{-10}$ m^2	N / A / 1.0	1.298×10^{-10} m^2	102111.39 $(1/A_0)$	13.3	0.000928	59
b (/MPa)	-2.948×10^{-6}	$\pm 8.35 \times 10^{-8}$ /MPa	R / A / 1.0	8.35×10^{-8} /MPa	70 (p)	5.9	0.000409	59
α_p (/°C)	4.55×10^{-6}	$\pm 4.55 \times 10^{-7}$ °C	R / B / $\sqrt{3}$	2.63×10^{-7} °C	1 $(T - T_r)$	0.27	0.000019	∞
α_c (/°C)	4.55×10^{-6}	$\pm 4.55 \times 10^{-7}$ °C	R / B / $\sqrt{3}$	2.63×10^{-7} °C	1 $(T - T_r)$	0.27	0.000019	∞
$T - T_r$ (°C)	1	±1 °C	R / B / $\sqrt{3}$	0.577 °C	9.1×10^{-6} $(\alpha_p + \alpha_c)$	5.3	0.000368	∞
P_N (MPa)	70	±0.00525 Mpa	N / B / 1.0	0.00525 MPa	-2.948×10^{-6} (b)	0.01	0.000001	∞
P_{DWT} (MPa)	70	±0.00525 Mpa	N / B / 1.0	0.00525 MPa	0.014286 $(1/P_DWT)$	75	0.0053	∞
$u_c(P_{DWT})$	$\sqrt{(u_{Type\,A})^2 + (u_{Type\,B})^2}$					77	0.0054	64961
Expanded Uncertainty (U)	70 MPa		$k = 2.0$	0.011 MPa				

3.2.6. Reporting of Results

For the pressure range of 0.5–70 MPa, the uncertainty associated with pressure measurements using the dead weight tester under reference is +0.011 MPa at a confidence interval defined by an expanded uncertainty $U = k\, u_c(P_{Test})$ and a coverage factor $k = 2$ based on Student's distribution for $\nu = 64961$ degree of freedom, and is estimated to a level of confidence of 95.45%. The detailed uncertainty budget thus prepared at a maximum pressure of 70 MPa using EAL Guidelines is shown in Table 3.

4. EVALUATION OF ASSOCIATED UNCERTAINTY USING NABL GUIDELINES

The ISO Guide (1995) [10] stipulates that the estimated variances and resulting standard uncertainties of an input quantity can be obtained from a curve that has been fitted to experimental values by the method of least squares through well-known statistical procedures. Based on this guideline, the NABL, India presents a method for evaluating uncertainty in pressure measurement using dead weight testers through statistical analysis of error and linear curve fitting [9].

4.1. Mathematical Modeling

As per NABL method, if P_{GUT} is the nominal pressure measured by the industrial dead weight tester, $\overline{P_{SPC}}$ is the average measured pressure of repeated measurements by the standard and ΔP is the difference in nominal and average measured pressure, then the mathematical relationship in this calibration is defined as:

$$P_{GUT} = \overline{P_{SPC}} + \Delta P, \tag{9}$$

where P_{GUT} is the pressure measured by the gauge under test, $\overline{P_{SPC}}$ is the average pressure obtained from the measured pressure values P_{DWT} by the standard DWT and ΔP is the difference between the two indications of P_{GUT} and $\overline{P_{SPC}}$. It is normally observed that ΔP is a linear function of $\overline{P_{SPC}}$, then we have:

$$\Delta P = \Delta P_0 + S_1 \overline{P_{SPC}}, \tag{10}$$

where ΔP_0 and S_1 are the constants of linear Eq. (10). Therefore, Eq. (9) becomes:

$$P_{GUT} = \overline{P_{SPC}} + \Delta P_0 + S_1 \overline{P_{SPC}}. \tag{11}$$

Table 4. Calibration Results Obtained.

Uncertainty evaluation for calibration of pressure dial gauge						
Weight Used on Test DWT	P_{GUT} (Nominal) (MPa)	$P_{DWT} = P_{SPC}$ (MPa)	$u(\overline{P_{SPC}})$ (MPa)	$\sigma(P_{SPC})$ (MPa)	$u_A = \sigma(P_{SPC})/\sqrt{n}$ (MPa)	ΔP (MPa)
W, 15	7	6.998447	6.998016	0.000244	0.000100	0.001984
		6.997836				
		6.99808				
		6.997842				
		6.998075				
		6.997813				
W, 2	10	9.99733	9.997060	0.000194	0.000079	0.002940
		9.996955				
		9.997278				
		9.996881				
		9.997				
		9.996918				
....	....	....	....	....	....	....
W, 2-12	60	59.9908	59.990735	0.000370	0.000151	0.009265
		59.99078				
		59.99037				
		59.99053				
		59.99052				
		59.99141				
W, 2-13, 15-17	70	69.99106	69.991222	0.000428	0.000175	0.008778
		69.99124				
		69.99099				
		69.99102				
		69.99095				
		69.99207				
W denotes piston plus carrier plus mass carrying bell						

The combined uncertainty is then given by:

$$u_c\left(P_{GUT}\right) = = \sqrt{\left[\left(\frac{\partial P_{GUT}}{\partial \overline{P_{SPC}}}\right) u\left(\overline{P_{SPC}}\right)\right]^2 + \left[\left(\frac{\partial P_{GUT}}{\partial \Delta P_0}\right) u\left(\Delta P_0\right)\right]^2 + \left[\left(\frac{\partial P_{GUT}}{\partial \Delta S_1}\right) u\left(\Delta S_1\right)\right]^2}, \quad (12)$$

where $\left(\frac{\partial P_{GUT}}{\partial \overline{P_{SPC}}}\right)$, $\left(\frac{\partial P_{GUT}}{\partial \Delta P_0}\right)$ and $\left(\frac{\partial P_{GUT}}{\partial S_1}\right)$ are the sensitivity coefficients of P_{GUT} with respect to $\overline{P_{SPC}}$, ΔP_0 and S_1, derived from partial derivation of Eq. (11) and are equal to 1, 1 and $\overline{P_{SPC}}$, respectively. therefore, we have:

$$u_c(P_{GUT}) = \sqrt{\left[u\left(\overline{P_{SPC}}\right)\right]^2 + [u(\Delta P_0)]^2 + \left[\overline{P_{SPC}}u(S_1)\right]^2}. \quad (13)$$

The uncertainty components $\overline{(P_{SPC})}u(S_1)$ and $u(\Delta P_0)$ are then evaluated through Type A method and the uncertainty associated with DWT, $u\overline{(P_{SPC})}$ is evaluated through Type B method. The calibration results for four exemplary pressure points are shown in Table 4.

4.2. Evaluation of Type A Standard Uncertainty

4.2.1. Repeatability

The standard uncertainty evaluated from the repeatability of the data from Table 4 as a linear function of pressure p is $u_{A1} = (0.0000037 + 1.6 \times 10^{-6} p)$ MPa.

4.2.2. Error Component

The error component is evaluated through linear least squares fitting of ΔP as a function of $\overline{(P_{SPC})}$ for all the 10 data points. The step by step computation is depicted in Table 5. The mathematical expressions for evaluation of fitting constants and their corresponding uncertainties are as follows:

$$\Delta P_0 = \left[\frac{\sum x^2 \sum y - \sum x \sum xy}{n \sum x^2 - (\sum x)^2}\right] = 0.002958 \text{ MPa}, \quad (14)$$

$$S_1 = \left[\frac{n \sum xy - \sum x \sum y}{n \sum x^2 - (\sum x)^2}\right] = 0.000112, \quad (15)$$

$$u(\Delta P_0) = \sqrt{\left[\frac{\{s^2 \sum x^2\}}{n \sum x^2 - (\sum x)^2}\right]} = 8.04 \times 10^{-4} \text{ MPa}, \quad (16)$$

$$u(S_1) = \sqrt{\left[\frac{\{ns^2\}}{n \sum x^2 - (\sum x)^2}\right]} = 2.09 \times 10^{-5}. \quad (17)$$

Table 5. Linear least squares fitting of the error component.

$x = \overline{(P_{SPC})}$	$y = \Delta P_0$	xy	x^2	ΔP_0	S_1	$y_{cal} = \Delta P_0 + S_1 x$	$(y - y_{cal})$	$(y - y_{cal})^2$	$s^2 = \Sigma(y - y_{cal})^2/n - 2$	u_{A2}
6.998016	0.001984	0.013888	48.972221	0.002928	0.000112	0.003709	−0.001724	2.9736E-06	1.62259E-06	0.000817
9.997060	0.002940	0.029388	99.941215			0.004043	−0.001104	1.2183E-06		0.000831
14.995613	0.004387	0.065781	224.868419			0.004601	−0.000214	4.5938E-08		0.000863
19.994332	0.005668	0.113335	399.773299			0.005159	0.000510	2.5985E-07		0.000906
24.993335	0.006665	0.166581	624.666794			0.005716	0.000949	9.0025E-07		0.000959
29.992270	0.007730	0.23184	899.536260			0.006274	0.001456	2.1206E-06		0.001019
39.991115	0.008885	0.355321	1599.289279			0.007389	0.001496	2.2377E-06		0.001160
49.990552	0.009448	0.472327	2499.055256			0.008504	0.000944	8.9087E-07		0.001318
59.990735	0.009265	0.555814	3598.888286			0.00962	−0.000355	1.2598E-07		0.001489
69.991222	0.008778	0.614406	4898.771110			0.010735	−0.001957	3.8302E-06		0.001669
Σx	Σy	Σxy	Σx^2	$u(\Delta P_0)$	$u(S_1)$			$\Sigma(y - y_{cal})^2$		
326.934249	0.065751	2.618681	14893.762140	8.04E-04	2.09E-05			1.460E-05		

Therefore, the uncertainty associated with error component is then evaluated as:

$$u_{A2} = u(\Delta P) = \sqrt{[u(\Delta P_0)]^2 + \left[\overline{P_{SPC}}u(S_1)\right]^2}. \quad (18)$$

As u_{A2} is the function of pressure p, the standard uncertainty evaluated due to error component as a linear function of pressure p from Table 5 is:

$$u_{A2} = (0.000657 + 13.65 \times 10^{-6} p) \text{ MPa}. \quad (19)$$

Therefore, the combined standard uncertainty evaluated through Type A method is the sum of root sum square of fixed and relative uncertainty components of u_{A1} and u_{A2}, separately:

$$u_A = \left\{ \sqrt{(0.0000037)^2 + (0.000657)^2} + \sqrt{(1.6 \times 10^{-6} p)^2 + (13.65 \times 10^{-6} p)^2} \right\} \text{ MPa} = $$
$$= (0.000658 + 13.74 \times 10^{-6} p) \text{ MPa}. \quad (20)$$

4.3. Evaluation of Type B Standard Uncertainty

The uncertainty contribution due the uncertainty reported in the calibration certificate of the DWT as $150 \times 10^{-6} p$ at a coverage factor $k = 2$ for the pressure range 0.2 to 100 MPa is then evaluated for normal distribution as $u_B = 75 \times 10^{-6} p$.

4.4. Combined Standard Uncertainty

The combined standard uncertainty $u_c(P_{GUT}) = (0.000658+76.25\times10^{-6}p)$ MPa is then evaluated form the sum of root sum square of all the fixed and relative uncertainty components evaluated through Type and Type B methods.

4.5. Effective Degree of Freedom of $u_c(P_{GUT})$

The effective degree of freedom calculated at 70 MPa is as follows:

$$\upsilon_{eff} = \frac{u_c(P_{GUT})^4}{\frac{(u_A1)^4}{\upsilon_A} + \frac{(u_A2)^4}{\upsilon_A} + \frac{(u_B)^4}{\infty}} = 386. \quad (21)$$

4.6. Expanded Uncertainty

Using the Student's table, $k = 2$ for a confidence level of approximately 95.45%, the expanded uncertainty is computed as $U = k\,u_c(P_{GUT}) = (0.00132+153.5\times10^{-6}p)$ MPa.

4.7. Reporting of Results

For the pressure range of 0.5–70 MPa, the uncertainty associated with pressure measurements using the dead weight tester under reference is ± ((0.00132 + 153.5 × $10^{-6}p$) MPa) (where p is pressure in MPa) at a confidence interval defined by an expanded uncertainty $U = k\,u_c(P_{GUT})$ and a coverage factor $k = 2$ based on Student's distribution for $v = 386$ degree of freedom, and is estimated to a level of confidence of 95.45%. The detailed uncertainty budget thus prepared at a maximum pressure of 70 MPa using NABL Guidelines is shown in Table 6.

Table 6. Uncertainty budget prepared at 70 MPa using NABL Approach.

Source of Uncertainty (X_I)	Estimates (x_I) (MPa)	Limits $\pm\Delta x_i$ (MPa)	Probability Distribution – Type A or Type B / Factor	Standard Uncertainty $u(x_i)$ (MPa)	Sensitivity Coefficient	Uncertainty Contribution $u_i(y)$ (MPa)	Degree of freedom (υ_{eff})
u_{A1} (Repeatability)	70	0.000175	Normal – Type A	0.000175	1	0.000175	5
u_{A2} (ΔP)	0.0022	0.0022	Normal – Type A	0.0022	1	0.0022	8
u_B (DWT)	70	0.0105	Normal – Type A	0.0053	1	0.0053	∞
$u_c(P_{GUT})$ Combined				0.0058	–	–	
Expanded Uncertainty			$k = 2$	0.0116	–	–	386

Note: u_B is the combined standard uncertainty evaluated using Type B method as $u_B = \sqrt{[u_{B1}^2 + u_{B2}^2]}$
The combined standard uncertainty associated with pressure measurements is then
$u_c(P_{GUT}) = \sqrt{(u_A)^2 + (u_B)^2}$
The expanded uncertainty is then $U = k\,u_c(P_{GUT})$

5. RESULTS AND DISCUSSIONS

The comparison of uncertainty thus estimated at a maximum pressure of 70 MPa using EAL and NABL approached is 0.011 MPa and 0.0116 MPa, respectively. It is clearly evident from the estimated uncertainties that both the estimated uncertainties are quite comparable and close. The uncertainty estimates through EAL approach do not take into consideration the indicated or nominal value of the measurand but these are obtained from the average value of the measurand. Even, if there is something wrong on the part of the manufacturers in assigning the nominal value, the uncertainty associated with measurements can be better or it may provide better-repeated results. The uncertainty estimates through NABL-141 document provide comparatively higher measurement uncertainty which is due to the fact that in this approach the individual systematic uncertainty contributions are considered to be part of the standard uncertainty error evaluated through curve fitting taking into consideration the relation between correction and the indicated or nominal value of the measurand and naturally become part of Type A evaluation. Since the method of least squares uses the square root of the average of the squares of the residual errors, the standard deviations of its constants and the standard uncertainty associated in the curve fitting become slightly higher. The other advantage of the NABL method is that it is a rather simple technique and can be used at the shop floor level or industry level.

6. CONCLUSION

A comparative study of the contemporary approaches, EAL and NABL, being used for evaluation of measurement uncertainty using dead weight piston manometer has been discussed through a case study. It is concluded from the study that the universal approach is the best way to predict comprehensive, globally accepted and reduced-uncertainty estimates. Although this approach is rather complex, tedious and time consuming, it is still the best technique to be used by the metrologists, scientists, engineers and accredited laboratory workers to analyze their results of national and international key comparisons, inter-laboratory comparisons, proficiency testing of the standards and for high precision measurements. Though one should always prefer the universal approach, the NABL-141 approach can also be used at the industry or shop floor level where it is not always possible to prepare a comprehensive uncertainty budget like production site etc.

REFERENCES

1. Heydemann P. L. M., Welch W. E.: *Experimental Thermodynamics*, vol. II, ed. by B. Leneindre and B.Voder , Butterworths, London, 1975.
2. Dadson R. S., Lewis S. L., Peggs G. N.: *The Pressure Balance: Theory and Practice*, Published by Her Majesty's Stationary Office, London, 1982.
3. Yadav S., Agarwal R., Bhanot S., Bandyopadhyay A. K., Gupta A. C.: *Modern Instrumentation Techniques in Pressure Metrology under Static Conditions*, Mapan: J. Metrol. Soc. India, 18, 57–82, 2003.
4. Yadav S.: *Pressure measurements: fundamental aspects*, Proc. of Seminar on Pressure Measurements and Calibration – It's Relevance to ISO 9000", Nov. 21, 1997, Bombay, pp. 1–11.
5. *Pressure Balances: Calibration*, Nordtest Method, NT MECH 009 2, 1987.
6. *Directive for the Calibration of Pressure Balances within the Scope of the German Calibration Service*, D.K.D.-R 3–4, June 1984.
7. *Pressure Balances*, OIML International Recommendation, OIML R110, 1994(E).
8. *Calibration of Pressure Balances*, EAL-G26, April 1997.
9. *Guidelines for Estimation and Expression of Uncertainty in Measurement*, NABL Document – 141, 2000.
10. *Guide to the Expression of Uncertainty in Measurement*, ISO Document – ISO/TAG/WG 3: 1995 (E).