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THE ANTIALIASING FILTER PARAMETERS PROPOSITION FOR LABORATORY AND DOMESTIC EQUIPMENT APPLICATION OF POWER QUALITY MEASUREMENTS SYSTEMS

Digital signal processing exists in systems which perform the processing of power waveforms. All measuring devices that use digital signal processing and computing of its discrete spectrum should be equipped with an antialiasing filter which, unfortunately, creates errors in magnitude and phase spectrum responses. This paper discusses the spectrum distortion analysis of the power signals caused by antialiasing filter implementation in the analog-digital signal processing system paths. The paper is a continuation of problems included in [1]. On the basis of the same research methodology the article completes research carried out in [1] about a real input signal model. Thus, it has been possible to describe antialiasing filter parameters in power quality measurement systems for laboratory and domestic applications.

Keywords: Antialiasing filters, power quality, digital measurements, signal processing

1. INTRODUCTION

Power quality monitoring issues are widely discussed nowadays. For the measurement of power quality parameters, digital systems are applied, including analog-to-digital converters. The analog-to-digital conversion signal paths have to be equipped with antialiasing filters which introduce additional errors to the measured signal magnitude spectrum. In [1], the minimization of aliasing errors with the help of proper selection of antialiasing filter parameters and sampling frequency is discussed. The manner of error evaluation in the case of modeling the input signal with the help of a white noise model is also presented there. It enabled to compute the maximum error level that can occur from the theoretical point of view. This situation does not practically exist in real power networks. In this article, the investigation is carried out in the same manner as in [1] and includes the input signal model. The researches enabled to describe error levels which may appear in real networks. On the basis of researches, antialiasing filter parameters and sampling frequencies for laboratory (considering a white noise input signal model) and domestic applications (considering a real input signal model), were proposed.

2. AN EXAMPLE OF THE ALIASING PHENOMENON APPEARING IN A REAL SIGNAL

The aliasing phenomenon is a commonly known subject and was described in detail in [2, 3] and in the author's recent paper [1].

The principle of aliasing creation is illustrated by the example of a real signal which has been measured in the power network of a hospital [1, 4, 5].

The representation of signal variation over time is presented in Fig. 1. The sampling frequency was 27280 Hz with 2728 samples. The normalized magnitude spectrum of the measured signal is illustrated in Fig. 1. For DFT computation a rectangular window was applied. For the paper purpose, the measured spectrum is assumed as a real signal spectrum. The example below is presented to give a general view of the aliasing phenomenon and antialiasing filtration problem.

Because there is noise within the signal, with a maximal amplitude of about 30 V, a wideband character of the signal can be observed. Moreover, the spectrum includes a local maximum at 10120 Hz and its value is equal to 2% of the fundamental.

In the case where the sampling frequency is downsampled to 13640 Hz, there is a significant probability that the new signal will be disturbed.

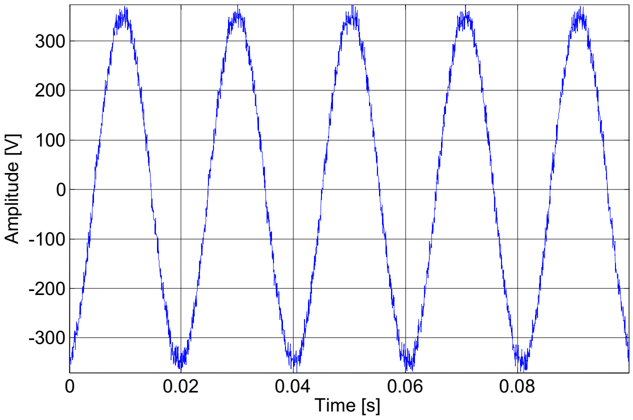

Fig. 1. Voltage signal in a real power network in the time domain: sampling frequency 27280 Hz, amount of samples 2728.

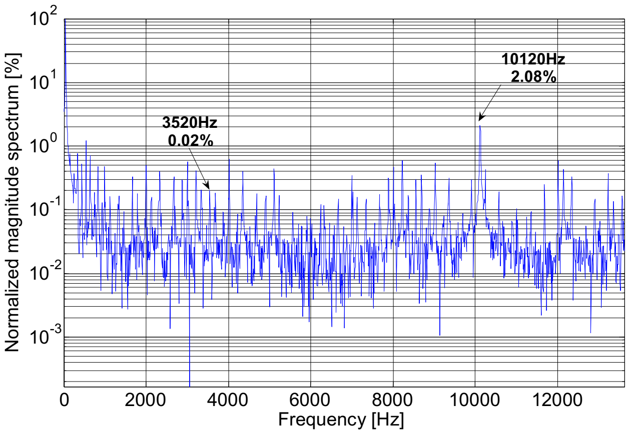

Fig. 2. Normalized magnitude spectrum of the signal: sampling frequency 27280 Hz, amount of samples 2728.

The 10120 Hz component introduces harmful interferences to the 3520 Hz component (13640 Hz – 10120 Hz). This situation is presented in Fig. 3, where the value of the 3520 Hz component has increased from 0.02% to about 2%.

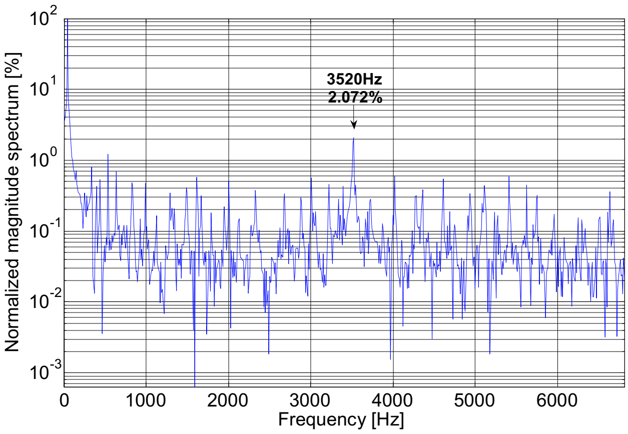

Fig. 3. Normalized magnitude spectrum of the signal which includes an error due to the aliasing phenomenon: sampling frequency 13640 Hz, amount of samples 1364.

In the example presented above, poor selection of the sampling frequency and lack of an antialiasing filter made a correct interpretation of the real magnitude spectrum impossible.

3. THE ANTIALIASING FILTER APPLICATION

To limit the influence of the aliasing phenomenon on the measured signal spectrum, low-pass antialiasing filters are applied between the conditioning circuit and the analog-digital converter [1, 2, 3, 6].

Let us choose a Butterworth-characteristic filter and a cutoff frequency of 4 kHz (proper filter parameters shown in 4.4 section are shown). In the case of filtering the signal presented in Fig. 1 the magnitude spectrum shown in Fig. 4 can be created.

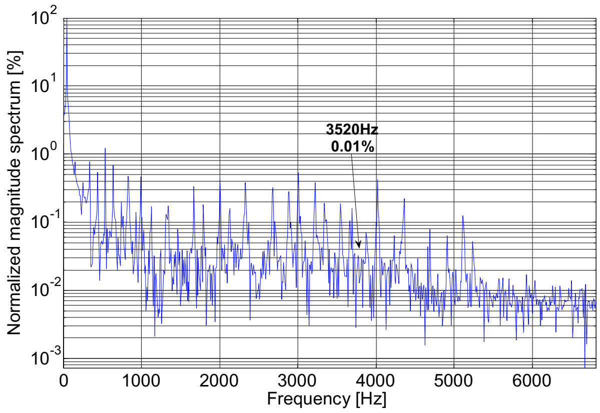

Fig. 4. Normalized magnitude spectrum of the filtered signal. Sampling frequency 13640 Hz, amount of samples 1364.

Thanks to signal filtration with the above cutoff filter frequency, the 10120 Hz component was removed and, in consequence, the aliasing phenomenon was limited. Therefore, the output spectrum at the frequency of 3520 Hz was decreased to about 0.01%.

The difference between the values of the component in question for the original spectrum and the down-sampled filtering spectrum rely on the fact that the filter for the frequency of 10120 Hz has a limited attenuation; it does not ensure full reduction of higher spectrum frequencies.

The influence of the aliasing phenomenon is described with the help of *Aliasing Error* (*AE*), Fig. 5. AE is the difference between the normalised magnitude spectrum of the real signal and the normalised magnitude spectrum of the signal after performing a downsampling operation (this spectrum is disturbed by the aliasing phenomenon).

The AE values appearing in the filtered and non-filtered signals are presented in Fig. 6. The maximum AE value included in the 0–6820 Hz band for the first case equals about 2%, whereas for the second case it is about 0.12%. The improvement of measurement accuracy is clearly visible here.

The values of individual harmonics in 0–2.5 kHz band are presented in Fig. 7.

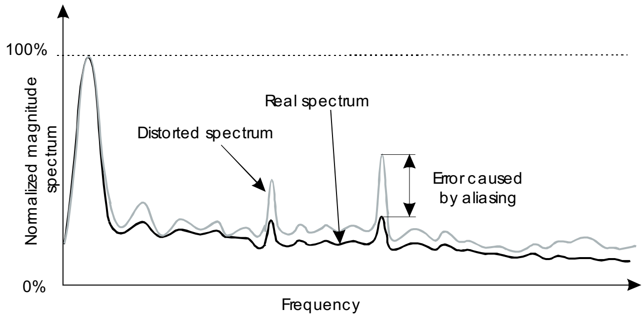

Fig. 5. The definition of Aliasing Error (AE).

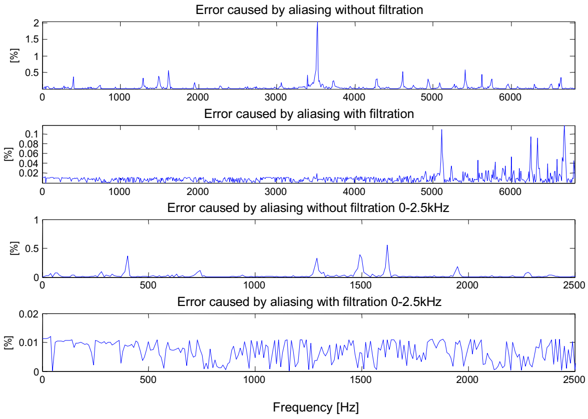

Fig. 6. The comparison of AE errors in the magnitude spectrum with and without aliasing filtration.

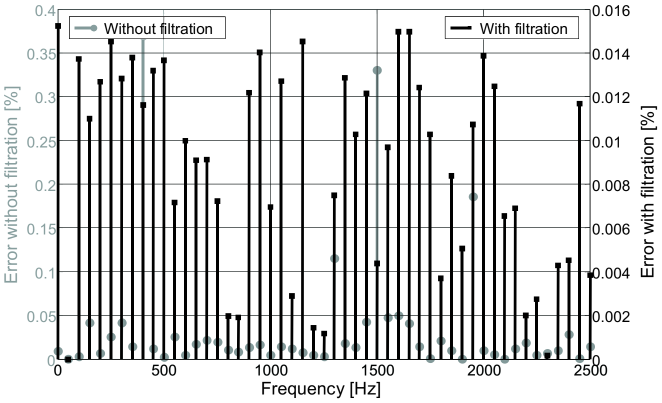

Fig. 7. The comparison of harmonic errors in band of 0–2500 Hz with and without aliasing filtration.

4. THE SELECTION OF FILTER AND SIGNAL PROCESSING PARAMETERS

4.1. Input parameters

The final selection of filter parameters and sampling frequency depends on the following input parameters:

- signal bandwidth, which may include a defined value of AE (aliasing error),
- maximum AE level in the defined band of signal,
- disturbance type appearing in the power network.

From the power signal analysis point of view, two main analysis groups are distinguished: low-frequency spectrum analysis up to 40 harmonics [7, 8, 9] and fast variable distortions. The main goal of the first kind of analysis is the calculation of the individual harmonics and interharmonics values, evaluation of the THD factor, monitoring of the flicker phenomenon etc. It gives good results when one is investigating signals with slowly-variable distortions. The width of the band needed for the analysis reaches 2.5 kHz. Fast-variable distortions, on the other hand, concentrate on rapid voltage changes, fast transient overvoltages, dips etc and the signal spectrum reaches 25 kHz.

Another input parameter taken into account while defining the antialiasing filter parameter refers to acceptable values of spectrum distortion in the analyzed bandwidth. For the purposes of this paper, an additional parameter, called the *width of the utilitarian band*, was defined [1]. This parameter defines a part of the pass-band where the error caused by the difference between the ideal and real filter magnitude response is maintained on the assumed level (i.e. 0.1%). In the paper, the results of research for utilitarian bands of 2.5 kHz and 25 kHz are presented.

The last parameter in question refers to the degree of power signal distortion. In most cases, the disturbances appearing in the power network have slowly-variable character. However, short-time duration disturbances also exist. The analysis of these kinds of disturbances requires faster analog-digital converters and more complicated filters. Therefore, it is important to propose values of the above parameters for domestic and laboratory equipment purposes.

4.2. The input signal models

Power signal measurement systems should provide correct information despite the distortions included in the measured signal.

In the critical case, the signal magnitude spectrum may remain equal in all bands. This case appears very rarely; however, it allows checking of the maximal error level [1]. Taking this input signal model into consideration guarantees that the measurement uncertainty will be smaller than computed, even in the most distorted real networks.

The signal model presented was defined as a broadband input signal model. This model was used during computations for laboratory power signal digital processing systems.

As mentioned above, a signal which may be approximated with the help of a broadband input signal model does not exist in practical cases. The aim then becomes to create a signal model which includes real disturbances occurring in real power networks. The time representation of such a model is presented in Fig. 8.

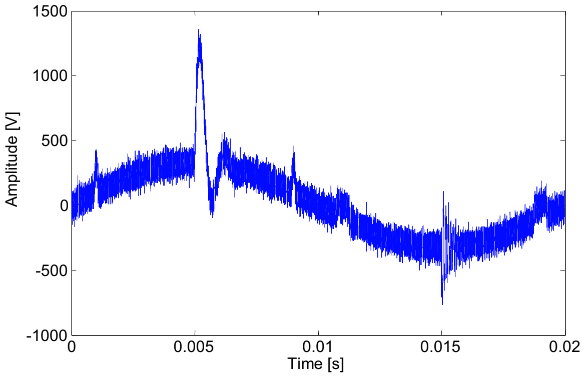

Fig. 8. The narrowband input signal model in the time domain (one period).

The model was created on the basis of measurements in hospital power networks. The model consists of two short-time sinusoidal exponential damped transient overvoltages of different amplitudes and frequencies, some dips and white noise.

The analysis of influence of the aliasing phenomenon on this signal type is presented below. The signal containing the proposed model is shown in Fig. 9.

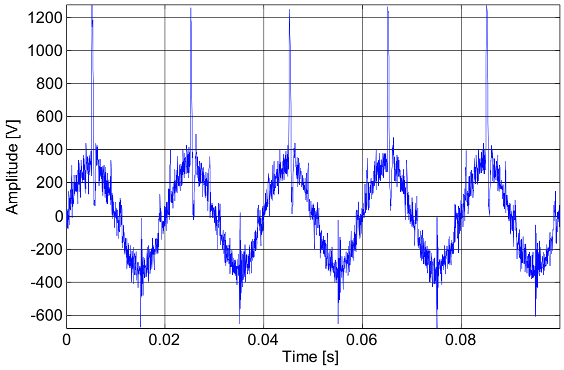

Fig. 9. The narrowband input signal model in the time domain (five periods).
Sampling frequency 27280 Hz, amount of samples 2728.

The magnitude spectrum of the signal in the frequency range up to 13640 Hz is presented in Fig. 10.

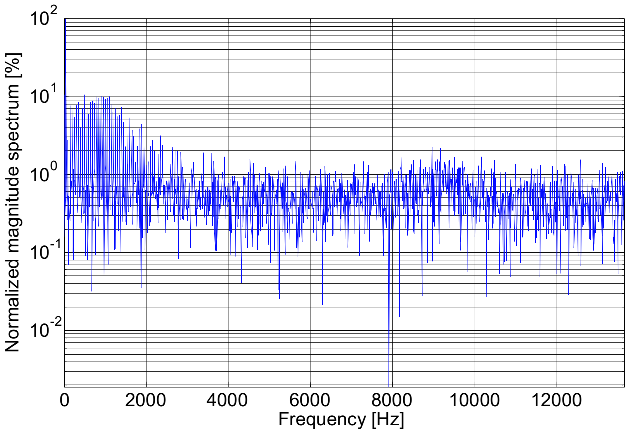

Fig. 10. Normalized magnitude spectrum of the signal presented on Fig. 9.
Sampling frequency 27280 Hz, amount of samples 2728.

An approximation of the magnitude spectrum of the signal from Fig. 10 (bold line in Fig. 11) enabled the creation of a test magnitude spectrum. The test spectrum was applied to evaluate filter parameters in domestic equipment. The signal represented by this kind of spectrum was called the narrowband input signal model (Fig. 11).

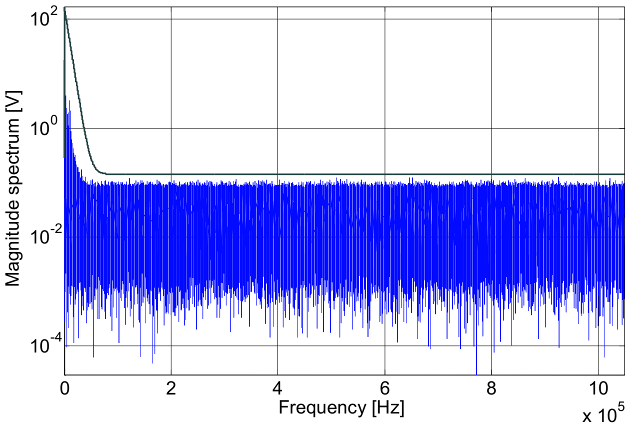

Fig. 11. The narrowband input signal model in the frequency domain and its approximation.

4.3. Total Output Error – TOE

To perform the design of an antialiasing filter and to specify its cutoff frequency fcut it is necessary to take into account the fact of representing 3-dB attenuation of the magnitude response. The higher harmonics in an input signal with a bandwidth reaching the cutoff frequency may be attenuated. The error inserted by an antialiasing filter will then equal about 30% for the cutoff frequency. To avoid this distortion it is necessary to correct the filter cutoff frequency. The new value should ensure the minimal attenuation in the whole utilitarian band and keep the error level in the assumed range. For further research, the error level was established as 0.1%. The filters which meet this condition will then insert no larger error to the signal spectrum. The characteristics in Fig. 12 presents hypothetical magnitude responses of 4th and 8th Butterworth filter orders before and after cutoff (fcut) frequency correction.

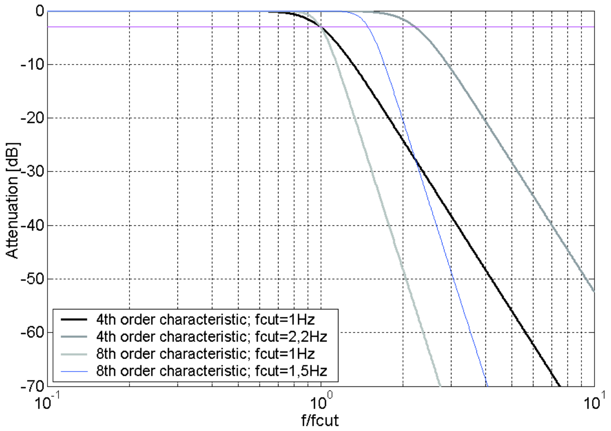

Fig. 12. The Butterworth filter magnitude responses of 4th and 8th order before (fcut = 1 Hz) and after correction (fcut = 2.2 Hz for 4th order and fcut = 1.5 Hz for 8th order) cutoff frequency.

In case of using the 4th Butterworth filter, which has to transmit 1 Hz bandwidth signal with a maximum error of 0.1%, a cutoff frequency of about 2.2 Hz should be applied. The cutoff frequency increases more then twice, then. There is also the influence of the filter order in this relation. For filters of the same type, but of the 8th order, the cutoff frequency should be 1.5 Hz. The relation between the cutoff frequency after and before correction may be defined as the *Cutoff Frequency Multiplier* (CFM) (1).

$$CFM = \frac{f_{cut\ after\ correction}}{f_{cut\ before\ correction}}, \tag{1}$$

where: $f_{cut\ after\ correction}$ – filter cutoff frequency ensuring a maximum error level established at 0.1%, $f_{cut\ before\ correction}$ – standard filter cutoff frequency defined as a 3 dB increase of attenuation.

The value of the CFM depends on filter order and type of filter characteristics.

The second factor which affects total measurement error of signal magnitude spectrum is the aliasing phenomenon. As mentioned above, this phenomenon may be reduced by increasing the sampling frequency or applying an antialiasing filter.

The error which consists of magnitude spectrum approximation accuracy and aliasing phenomenon, was defined as Total Output Error – TOE.

4.4. The results of the research

The subject of the research, similarly as in [1], includes low-pass filters of 1st to 10th order for the following characteristics types:

- Butterworth,
- Bessel,
- Chebyshev: utilitarian band ripple 0.0087 dB (0.1%),
- elliptic: utilitarian band ripple 0.0087 dB (0.1%), stop-band ripple 10 dB, 20 dB, 30 dB, 40 dB, 50 dB and 60 dB.

The CFM values as a function of filter order, characteristic type and AE value are presented in Table 1 and 2 (in [1] the CFM values were computed only for characteristic error of 0.1%). The CFM values above the 10th order are practically stable.

Table 1. The CFM values for TOE error below 0.1%.

Filter order	Filter type								
	Butterworth	Bessel	Chebyshev	Elliptic 10dB	Elliptic 20dB	Elliptic 30dB	Elliptic 40Db	Elliptic 50dB	Elliptic 60dB
1	22.73	22.73	22.22	22.22	22.22	22.22	22.22	22.22	22.22
2	4.74	17.86	3.41	2.99	3.28	3.37	3.40	3.41	3.42
3	2.82	17.86	1.92	1.56	1.73	1.82	1.88	1.89	1.91
4	2.18	18.18	1.49	1.20	1.30	1.37	1.42	1.45	1.47
5	1.86	18.18	1.31	1.08	1.13	1.18	1.22	1.25	1.27
6	1.68	18.52	1.22	1.03	1.06	1.09	1.12	1.15	1.17
7	1.56	18.52	1.15	1.01	1.03	1.05	1.07	1.09	1.10
8	1.47	18.52	1.12	1.01	1.02	1.03	1.04	1.06	1.07
9	1.41	18.52	1.09	1.00	1.01	1.02	1.02	1.03	1.04
10	1.37	18.52	1.07	1.00	1.00	1.01	1.01	1.02	1.03

Table 2. The CFM values for TOE error below 1%.

Filter order	Filter type								
	Butterworth	Bessel	Chebyshev	Elliptic 10dB	Elliptic 20dB	Elliptic 30dB	Elliptic 40Db	Elliptic 50dB	Elliptic 60dB
1	7.04	7.04	7.04	7.04	7.04	7.04	7.04	7.04	7.04
2	2.65	5.59	2.00	1.78	1.93	1.98	2.00	2.00	2.00
3	1.92	5.56	1.41	1.21	1.30	1.36	1.39	1.40	1.41
4	1.63	5.65	1.23	1.06	1.12	1.16	1.18	1.20	1.21
5	1.48	5.75	1.14	1.02	1.05	1.07	1.10	1.11	1.12
6	1.39	5.78	1.10	1.01	1.02	1.04	1.05	1.06	1.07
7	1.32	5.78	1.07	1.00	1.01	1.02	1.03	1.04	1.04
8	1.28	5.81	1.05	1.00	1.01	1.01	1.02	1.02	1.03
9	1.24	5.81	1.04	1.00	1.00	1.00	1.01	1.01	1.02
10	1.22	5.85	1.04	1.00	1.00	1.00	1.00	1.01	1.01

The research methodology of antialiasing filter parameters and sampling frequency (up to 1 MHz) selection is applied similarly as in [1].

The filter parameters and sampling frequencies for the following equipment were evaluated:

- laboratory equipment for:
  - TOE: 0.1%, 0.5% and 1%,
  - measurements of long-time duration distortion (utilitarian band equals 2.5 kHz) and measurements of short-time duration distortion (utilitarian band equals 25 kHz),
- domestic equipment for:
  - TOE: 1%, and 10%,
  - measurements of long-time duration distortion (utilitarian band equals 2.5 kHz) and measurements of short-time duration distortion (utilitarian band equals 25 kHz).

The antialiasing filter parameters and sampling frequency selection criteria for the first type of systems assumed minimalization of errors introduced to the measured signal spectrum (broadband input signal model) and minimalization of signal distortion in the time domain. This assumption influences the increase of filter cutoff frequency and the sampling frequency applied to the system. However, such an approach significantly increases the costs of the system. The filter parameters for TOE values of 0.1%, 0.5% and 1% for laboratory equipment are presented in Fig. 13.

Because of the need to minimize domestic equipment costs, two assumptions were accepted in the calculations: applying the narrow-band input signal model, and using a CFM value which corresponds to a deviation of the real characteristic from the ideal at the 1% level. This obviously influences the measurement accuracy and equipment sensitivity for broadband distortions, atypical but seldom occurring in real power networks. In Fig. 14, the filter parameters and minimum sampling frequencies are presented for TOE of 1% and 10%.

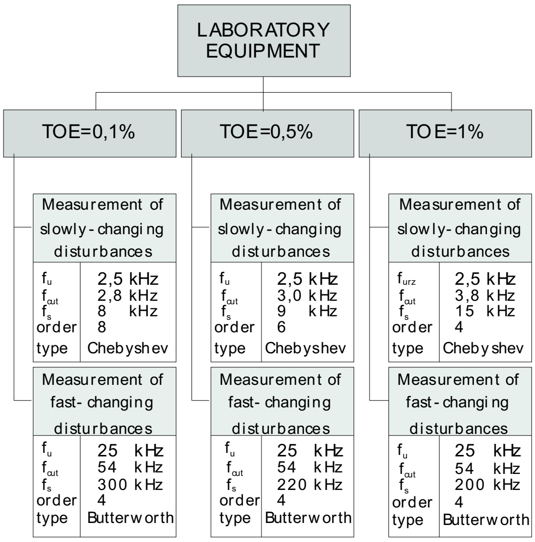

Fig. 13. The decision algorithm of filter parameters and sampling frequency selection for laboratory equipment.

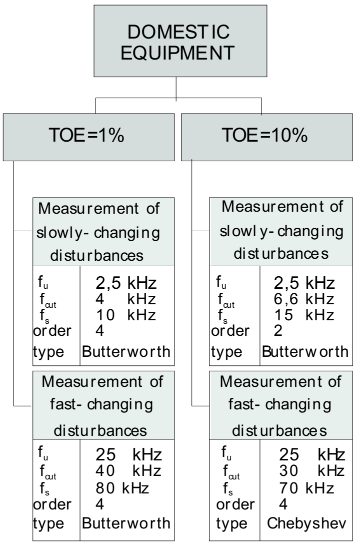

Fig. 14. The decision algorithm of filter parameters and sampling frequency selection for domestic equipment.

Application of an antialiasing filter has also an influence on the phase spectrum of the measured signal. It appears particularly for higher-order filters and for frequencies near the cutoff frequency. Because of this phenomenon, the filter cutoff frequency should be properly higher than the highest frequency in the filter utilitarian band, and filter order should not exceed the 8th. For example, in the case of a filter for domestic applications (Fig. 14), for measurements of slowly-changing disturbances (utilitarian band equal to 2.5 kHz) and TOE value of 1%, 4th order Butterworth filters with a cutoff frequency of 4 kHz and sampling frequency 10 kHz are proposed.

5. CONCLUSION

The influence of the application of an antialiasing filter on the signal spectra measured by equipment with the use of analog-digital conversion was presented in this article. The influence of the aliasing phenomenon and the antialiasing filter characteristic type were taken into consideration. The uncertainties of individual harmonics with and without the application of an antialiasing filter were presented. In the article, analyses of the influence of filter's existence for different filter orders, cutoff frequencies and approximating polynomial types were performed. A set of optimal filter parameters, dependent on measured distortion type, maximal error level introduced to the signal magnitude spectrum, and application area of the system (laboratory equipment, domestic equipment), was proposed. The minimum sampling frequencies which have to be applied to the system were presented. In the evaluations, both a theoretical model of signals and models built on the basis of measurements in real power networks were taken into consideration. The first of them enables the definition of filter parameters whose application guarantees compensation of the aliasing error at a defined level, even in very distorted power networks. The second model, on the other hand, enables the specification of filter parameters whose application allows the measurement of the signal in power networks in the presence of the most frequently occurring disturbances.

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