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OPTIMAL MEASUREMENT POLICY FOR DECISION MAKING: A CASE STUDY OF OUALITY MANAGEMENT BASED ON LABORATORY MEASUREMENTS

Measurement information generates value, when it is applied in the decision making. An investment cost and maintenance costs are associated with each component of the measurement system. Clearly, there is – under a given set of scenarios – a measurement setup that is optimal in expected (discounted) utility. Contrary to process design, design of measurement and information systems has not been formulated as such an optimization problem, but has rather been tackled intuitively. In this presentation we propose a framework for analyzing such an optimization problem. Our framework is based on that the basic mechanism of measurement is reduction of uncertainty about reality. Statistical decision theory serves as the basis for analyzing decision making. In this article we apply the framework to a problem that is rather simple but of practical importance: how to arrange laboratory quality measurements optimally. In particular, we discuss a case in the paper making industry, in which the product quality is measured with automated quality analyzers and by laboratory measurements.

Keywords: decision support, design, optimization, uncertainty, Bayes

1. INTRODUCTION

The process industries make use of hundreds of on-line and laboratory measurements to monitor and control the process [1]. Information systems are designed with the aim of supporting the daily decision making about process and product quality by operators and engineers so that the best practice of operation can be achieved continuously. Measurements, soft sensors and process simulators form the basis for such decision support by reducing the uncertainty about the present state of the process and about its future evolution.

Information derived from the process is used in many ways, but it is poorly known how and if the operators exploit all the information available. This may lead to a situation where some measurements are carried out without purpose, only by habit. However, if end user information requirements and constraints on uncertainty of the measurements are made explicit, the optimal arrangement of the measurements can be determined. [2]

When making decisions or when combining information from various sources, the uncertainty of information is decisive and must thus be known. Ideally measurements, soft sensors and simulators for estimation and forecasting should produce as their information output the probability density function of the state of the process and of its predicted evolution, respectively. In present information systems such uncertainties of estimates and forecasts are not recorded, and rather often the uncertainty analysis has been neglected altogether.

Information from measurements, soft sensors and simulators generates value through improved decisions [3], because the uncertainty about the present and/or future state of the process has been reduced. The amount of value generated depends on the goal set by the decision maker, including the decision maker's attitude towards risk. Process operators and engineers are rather unfamiliar with the concept of uncertainty and hence uncertainty of consequences is dealt with rather implicitly when making decisions [4].

The optimal measurement system is such that it maximizes the value of information generated, under a given set of scenarios on external effects to the process. Optimal process

design is well known [5], but the optimal design of information systems – measurements, actuators, control algorithms and data analysis methods – has emerged only recently [6-8].

The goal of this study is to keep uncertainty of state information below a pre-specified level while minimizing the costs of measurements. In this article, we expect the allowable level of uncertainty given, e.g. by analyzing the system decision tasks and performance requirements. This article tackles the optimal measurement policy problem with Bayesian approach and two case examples (passive and active) are presented. In the passive case a fixed set of quality measurements are always made and an output model gives the estimated values for the other quality parameters of interest. In the active case the monitoring of the estimated quality parameters determines dynamically when and which quality measurements of the measured set are to be made.

This article is organized as follows. In Section 2 we discuss statistical decision theory briefly as it is the framework within which we analyze value that information generates. Furthermore, we discuss the generic problem of measurement setup and propose that the relationship to optimal decision making is via specifying, how accurately the process state - in our case product quality - must be known. Section 3 discusses a practical way of designing the measurement setup and how that relates to optimal estimation in linear/Gaussian case. In Section 4 we formulate a case of papermaking in which the task is to find an optimal laboratory measurement scheme to support quality management and discuss this case with real-life data. Both the active and passive approaches are presented. In Section 5 we discuss how our results can be generalized to other cases in process industries. This section also explores the possibilities of future research.

2. STATISTICAL DECISION THEORY

Decisions are based on available information about the target system – current measurements, a priori information in the form of models and tacit knowledge. Decision making can be described as an optimization task, either a deterministic, stochastic, multigoal or game problem.

The formal statistical decision making problem consists of the following elements: a priori information about the state of the system, models of measurements, model for predicting the consequences of decision alternatives, and the expectation value of utility of the consequences. To define these elements, the system state (x), the set of consequences (c) and the set of allowable decisions (actions, a) must be described. Note that x, c and a are multidimensional and that they may be past time series (x) or future time series (c, a). Figure 1 presents the decision making task: given the measurement value $x^{(obs)}$ and the probabilistic models, what is the action that yields maximal expected utility for the decision maker (DM) [9, 10].

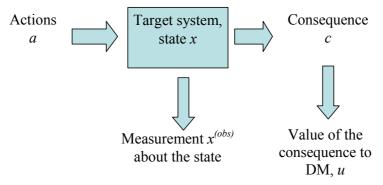


Fig. 1. Action - consequence scheme of system.

Decision maker knows the state of the system, x, only probabilistically through uncertain measurements and possibly through a priori information. The consequence c of the action a, given that system state is x, is known probabilistically as a priori information. DM evaluates the system performance in terms of consequences. The utilities of consequences c, if the consequence were certain, are given as u(c) [11]. Then the best action a^* is the one with highest expected utility. The utility is a description of both DM's preference order and attitude towards risk. If utility exists, DM is guaranteed rational in the sense that he does not have circular preferences in pairwise comparisons of decision alternatives.

Formally, the elements of a priori information, measurement models and prediction models are then, respectively, the probability density functions:

$$f_X^{(ap)}(x), (1a)$$

$$f_{X^{(obs)}|x}^{(meas)}(x^{(obs)}|x),$$
 (1b)

$$f_{C\mid a,x}^{(pred)}(c\mid a,x). \tag{1c}$$

Here $x^{(obs)}$ refers to the measured value of x. The probability density function of consequence c, given that $x^{(obs)}$ has been measured and DM would decide a is then according to Bayes formula [12, 13]:

$$f_{C|a,x^{(obs)}}^{(pred)}(c \mid a, x^{(obs)}) = N * \int_{domain(X)} f_{C|a,x}^{(pred)}(c \mid a, x) f_{x^{(obs)}|x}^{(meas)}(x^{(obs)} \mid x) f_{x}^{(ap)}(x) d^{n}x,$$
 (2)

where N is a normalization factor and n is the dimensionality of system state space description.

Defining the objective of decision making, and in particular the attitude towards risk, is quite often the main challenge when applying the formal decision theory to operational decision making about production, e.g. in papermaking and in other industrial processes. Although the utility function exists for a rational decision maker, its most general identification method through finding certainty equivalents of "gambling cases" [11] is tedious and often not intuitive for the decision maker. We shall employ the utility function as a normative decision model and assume that DM is able to express it notwithstanding that it has been criticized for not corresponding to human decision making in all respects [10, 14].

The optimal decision is then the one that maximizes the expected utility, and the corresponding expected utility is the measure of performance [8-10, 15]:

$$a^{*}(x^{(obs)}) = \underset{a \in A}{\arg \max} \int_{domain(C)} u(c) f_{C|a,x^{(obs)}}^{(pred)}(c \mid a, x^{(obs)}) d^{m}c$$

$$U^{*}(x^{(obs)}) = \int_{domain(C)} u(c) f_{C|a^{*}(x^{(obs)}),x^{(obs)}}^{(pred)}(c \mid a^{*}(x^{(obs)}),x^{(obs)}) d^{m}c$$
(3).

The properties of measurement system affect the optimal expected utility $U^*(x^{(obs)})$ through how accurately the state is known on the basis of measurement value $x^{(obs)}$. Assuming that there is no measurement bias, the properties of measurement system can be predominantly characterized by C_{xx} , the covariance matrix of measurement uncertainties. Hence we may write $U^*(x^{(obs)}, C_{xx})$.

In order to achieve an accuracy C_{xx} a (life-time, discounted) cost $c(C_{xx})$ is caused. When designing a measurement setup, we assume that there will be a number of decision making situations, each with its specific action-consequence model and utility. A scenario occurs with frequency p_i , and the corresponding optimal utility, if measurement $x^{(obs)}$ is made, is $U_i^*(x^{(obs)})$. Furthermore, for each scenario, we can assess the a priori probability density function of observing $x^{(obs)}$ to be $f_i(x^{(obs)})$. Then the design problem reduces into finding the optimal measurement accuracy maximizing the lifetime "profit":

$$C_{xx}^{*} = \underset{C_{xx}}{\operatorname{arg\,max}} \left[T * \left(\sum_{i=1}^{I} p_{i} \int_{domain(X)} U_{i}^{*}(x^{(obs)}, C_{xx}) f_{i}(x^{(obs)}) d^{n} x^{(obs)} \right) - c(C_{xx}) \right], \tag{4}$$

where *T* is the life time of the system and we have assumed that utility has been expressed in units comparable to those of costs.

Here we have considered only the direct effect of measurement setup (and accuracy) on value generated. Quite often the prediction model (1c) is identified and updated on the basis of the very same measurements. The better the accuracy of measurements the more accurate are the models and the better optimal utility in (3) can be achieved. Similarly, the a priori information about system state is based on long term statistics of the same measurements: the more accurate the measurements, the more accurate the a priori information and the better the decisions. The accumulating nature of this accuracy leads to complex discounting questions. Hence, we choose to neglect these indirect effects throughout the rest of the paper and concentrate on the direct effect only.

The measurement setup optimization described in (4) is extremely difficult to carry out in practice. We need to specify all decision making situations to arise during the lifetime of the system, their frequencies, and the utilities and prediction models associated with them. However, we should bear in mind that a similar analysis is the basis of optimal process design and should thus be the goal of optimal measurements system design as well.

3. MINIMUM-COST MEASUREMENT SETUP FOR SPECIFIED ACCURACY OF STATE INFORMATION

The analysis above shows that optimizing the measurement setup is equivalent to finding the optimal measurement accuracy, as described by C_{xx} . If we cannot solve the formal optimization problem (4), we may first seek to find "largest acceptable uncertainty" in the information about the system state through process expertise and then analyze by which measurement setup the uncertainty will be below this at lowest cost.

When estimating the system state the correlations between system state variables can be used to reduce the number of measurements to be made and hence to reduce measurement costs. The elements in the measurement setup analysis are depicted in Fig. 2.

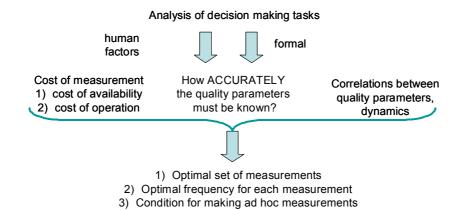


Fig. 2. Measurement setup analysis scheme.

Next we shall consider the following case

- a priori information: system state is multivariate normally distributed, $X \sim N_n(x; \mu, \Sigma)$; no other a priori information,
- measurement description: all state variables can be measured, the measurements are unbiased and distributed according to $X^{(obs)} \mid x \sim N_n(x_{obs}; x, C)$; the description of any subset of measurements is obtained by marginalizing the full distribution with respect to the measurements not made.

Let us assume that we have analyzed the tasks that DM will be facing assisted with the measurement information system. With human experts we have concluded that the quality of decision making requires that at all instances the largest allowable uncertainty in state variable x_i is $\sigma_i^{(c)}$:

$$\left(\sum_{xx}^{(post)}\right)_{ii} < \left(\sigma_i^{(c)}\right)^2 \tag{5}$$

and we have chosen not to limit the off-diagonal elements of $\Sigma_{xx}^{(post)}$, other than $\left|\left(\Sigma_{xx}^{(post)}\right)_{ij}\right| < \sigma_i^{(c)}\sigma_j^{(c)}$, guaranteed by (5). In order to satisfy (5), we may choose to measure once or several times some of the quality parameters and to estimate the other on the basis of the measurements made and the a priori correlations between the quality parameters. Such a measurement will be referred to as "measurement decision", and a sequence of measurement decisions a "measurement policy". We consider policies in which the measurement decisions are made at regular intervals. In the present case the measurement decisions will be independent of the actual values of measurements. Assuming we know the cost associated with each measurement decision, we then may solve for optimal decision policy.

When the a priori joint probability density function of system state is multivariate Gaussian, we know from optimal estimation theory [16] that dividing the quality parameters into two groups, $x = [x_1 \ x_2]$ and measuring x_1 with measurement errors having a multivariate normal joint probability density function $X_1^{(\text{obs})} \sim N_d(x^{(1)}, C_{11})$, the estimates for x_2 and the covariance matrix describing their uncertainties are given by

$$\begin{split} \hat{x}^{(1)} &= x_1^{(obs)} \\ \Sigma_{11}^{(post)} &= C_{11} \\ \hat{x}^{(2)} &= \mu_2 + B(x_1^{(obs)} - \mu_1) \\ B &= \Sigma_{21} \Sigma_{11}^{-1} \\ \Sigma_{22}^{(post)} &= \Sigma_{22} - \Sigma_{21} (\Sigma_{11}^{-1} - \Sigma_{11}^{-1} C_{11} \Sigma_{11}^{-1}) \Sigma_{12} \end{split}$$
 (6)

Here Σ_{11} is the submatrix of Σ_{xx} for variable set x_1 , and respectively for other Σ_{ij} and μ_i .

The diagonal elements of the covariance matrix of estimates, $\Sigma_{ii}^{(post)}$, give the left hand side of Eq. (5). Then we may proceed to solve for the lowest-cost laboratory setup satisfying the constraint of Eq. (5). If we repeat some of the measurements of $x^{(1)}$, this affects only the matrix C_{11} in the analysis above. Therefore measurement setups of repeats are also solved with the same approach, or formally:

$$[k]^* = \underset{[k]}{\operatorname{arg\,min}} c([k])$$

$$s.t.$$

$$[k] = (k_1, ...k_n); \quad k_i = 0,1,...$$

$$(\sum_{xx}^{(post)}([k]))_{ii} < (\sigma_i^{(c)})^2; \forall i$$

$$(7)$$

with n the dimension of the state and k_i of the repeats of measurement i. This solves for the static measurement decision: which measurements need to be made in order to confirm with the constraint of largest acceptable uncertainty about the state.

Obviously, in real-life applications we cannot trust the joint probability density function of quality parameters to be Gaussian and methods of nonlinear estimation need to be applied. However, following the principle outlined above.

The analysis above solved which measurements are to be made when only a priori information was the statistical dependence between the state variables. Therefore, it does not give us any information about how often the measurements are to be made. In process management, we have on the one hand the requirement that condition (5) must be satisfied at all times and on the other hand we have the additional a priori information from previous measurements and the state estimate based on those. The information based on earlier measurements degrades, and thus a need for new measurement arises once this information no longer satisfies the condition (5).

The Equations (6) provide the uncertainty of the estimate immediately after the measurement is made. It is intuitively obvious that as time progresses and no new measurements are made, the estimate can still be considered as the estimate of the process state, but the uncertainty increases with time. The approach of the maximum entropy principle is to assume that the process state undergoes a Wiener process (random walk) so that the estimate uncertainty increases in time as [17-20]:

$$\left(\sum_{xx}^{(post)}\right)_{ii}(t_n + \tau) = \left(\sum_{xx}^{(post)}\right)_{ii}(t_n) + D_{ii}\tau
0 < \tau < t_{n+1} - t_n$$
(8)

where t_n is the instant when the *n*th measurement/estimation was made and D_{ii} is the covariance parameter of the Wiener process of process state.

With the assumption of Eq. (8), estimation method (e.g. Eq. (6)) and constraints, Eq. (5), we may formulate an optimization problem: which measurement we need to make and how often (interval [0, T], present time is 0) to keep the knowledge about the quality within the required accuracy (cf. Fig. 2):

$$\begin{aligned}
&([k](t))^* = \underset{\{[k](t)\}_{t=0}^{T}}{\min} c([k](t)) \\
s.t. \\
&[k](t) = (k_1, ...k_n); \quad k_i = 0,1, ... \\
&t \in [0,T] \\
&(\sum_{xx}^{(post)}([k])_{ii} + D_{ii}\tau < (\sigma_i^{(c)})^2; \forall i, \forall t
\end{aligned} \tag{9}$$

Although defining the constraint for uncertainty (5) is extremely challenging for practical decision makers and the cost structure of making measurements may be much more complicated than each measurement having its cost independent from possible other measurements made at the same time, we claim the approach practical. We shall now proceed in applying the approach to the analysis of quality management and related laboratory activities at a paper mill.

4. QUALITY MANAGEMENT AT A PAPER MILL

In process industries such as papermaking the quality management is commonly based on a three-level hierarchical measurement structure: accurate but costly and infrequent laboratory measurements, automated quality analyzers sampling more frequently and mimicking laboratory analyses, and indirect but frequent on-line measurements for automatic control. In paper mills, measuring frequency of analyzers is usually once per machine reel, or 1-3 times an hour, whereas laboratory analyses are made at most 3 times a day. These frequencies are to be compared with that paper web is produced at web speed of up to 30 m/s, or 50 tons/h. The decisions supported with this information can be divided into three categories: continuous process and quality management, special actions, and configuration of the measurement information system itself.

At paper mills the active quality control is broke management and apportioning raw materials,. The most important special action in quality management is detecting off-specification products to be rejected. The validation and calibration of on-line quality sensors with other quality measurements is the main configuration decision. The accuracy constraints for quality information, Eq. (5), can be derived from analysis of these decisions: how data is actually used in each of the cases. There are only very few mills that have carried out such an analysis, and no mills that have applied decision analysis to specify how accurately the measured parameters must be known.

As a particular case we consider the management of the key optical properties of paper: brightness, opacity and L-a-b color coordinates. The optical quality specifications of printing paper grades set by customers are tight as the visual appearance of printed products hinges on these quality parameters. The optical properties are well standardized and there exist laboratory devices of high accuracy to measure these parameters. However, such laboratory activities are labor intensive and also require investing in devices and systems. Laboratory measurements can never be made with a frequency to give a sufficient understanding of optical quality variations within a customer reel. These properties can be measured with automated quality analyzers that have investment costs of similar magnitude but the

operational costs per sample are much lower. The practices of combining laboratory analyses and automated quality analyzers have been developed with time into their present form, and it may be questioned whether they are close to optimal. Should a green field production line be built, how should the laboratory activities be set up?

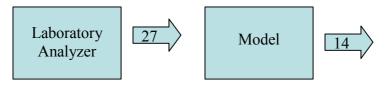


Fig. 3. Passive measurement policy.

The optimal measurement setup of optical quality in paper is based on analysis of quality measurement data from a paper mill, over a six-month period from both an automated analyzer (27 quality measurements) and laboratory analysis (14 quality parameters). The analysis concentrated on one paper grade only. There are two cases discussed in this paper; passive (Fig. 3) and active (Fig. 4) case measurement policies. In the passive case the cost of availability and operation for laboratory analyzer is the same regardless of the amount of quality parameters measured. So, all 27 quality measurements are measured though only few of them would be needed. In the active case every quality measurement costs the same but they can be measured separately. Laboratory analysis is labor intensive and the cost of availability and operation varies between quality parameters. The target is to find out if laboratory analyses are needed at all, and if they are needed, how often they must be made to maintain the requested accuracy of optical quality information.

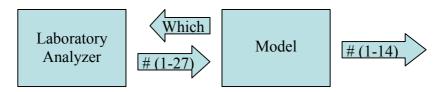


Fig. 4. Active measurement policy.

Following linear optimal estimation we divide quality parameters into two groups $x = [x_1 \ x_2]$ and measuring x_2 generates the estimates for x_1 . Every quality parameter in x_1 has the largest allowable uncertainty (constraint of Eq. 5) and every quality parameter in x_2 costs one unit if measured. In the passive case the cost is constant, 27 units, every time.

The objective was to find correlating quality parameters between these two measurement methods – laboratory analysis and laboratory analyzer – and to find the group of quality parameters that can be estimated using measurement results from a laboratory analyzer only, thus reducing the laboratory work. After that, labor intensive laboratory analysis can be focused on those quality parameters that cannot be estimated using this model and are needed.

Quality parameter data was divided into two parts; the first part (three months) was used as identification data for establishing the a priori model, and the second part (the next three months) as validation data. Identification data was used to generate models in form of covariance matrices and to find the optimal set of measurements. In the passive case all 27 of laboratory analyzer quality measurements were used in estimation, while in the active case the optimal set of measurements was defined for every quality parameter separately with stepwise analysis. The stepwise procedure was modified such that analysis was aborted when the R²-value reached a value of 0.865. The temporal optimization is valid till the next measurement needs to be made and can thus be called a one-step-ahead procedure. The length of the step is determined by the uncertainty of the estimate, that is, the model needs to be dynamically

validated via new laboratory analysis measurements when an uncertainty reaches the largest allowable uncertainty. Figure 5 shows measured and estimated values and errors between them for the quality parameter 1, using validation data set, that is, the measured value comes from laboratory analysis measurement and the estimated value is calculated with the model based on laboratory analyzer measurements. There is also a mean square error calculated for estimates. Figure 6 shows the very same information for the quality parameter 2. In the passive case estimations are estimated using all 27 of quality measurements, while in the active case only four measurements are used estimating the quality parameter 1 and five with the quality parameter 2.

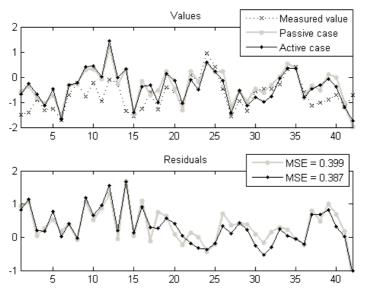


Fig. 5. Measured and estimated values (upper figure) and residuals as well as mean square error (lower figure) for quality parameter 1.

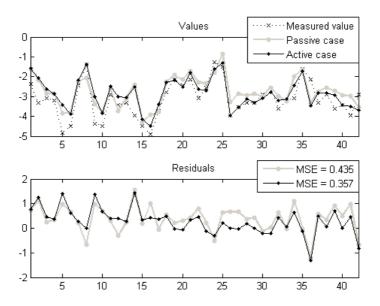


Fig. 6. Measured and estimated values (upper figure) and residuals as well as mean square error (lower figure) for quality parameter 2.

There is a covariance matrix of Wiener process, D in (Eq. 8), associated with quality information, so the estimate uncertainty increases in time. To maintain the uncertainty of quality information within an acceptable level, we choose D at its worst-case value, i.e. consider how quickly at most we may loose information about quality. A chosen set of the

quality measurements is measured and the model is dynamically validated every time the uncertainty of an estimate increases over $\sigma_i^{(c)}$. Figures 7 and 8 display the estimates and their uncertainties for a ten-day period for quality parameter 1 and 2, respectively. Solid black lines represent the estimate uncertainty with the worst-case D, grey lines the largest allowable uncertainty, circles represent measured and crosses estimated values and the black dotted line represents the target value. An apparent asymmetry in the estimate uncertainty occurs from variation of the estimate. The degraded laboratory information and the information based on analyzer results can be fused to reduce the number of laboratory analyses.

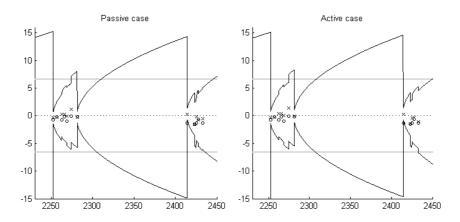


Fig. 7. Estimated values (cross), measured values (circle), uncertainties (solid black lines) and goals (grey line for largest allowable uncertainty and dotted line for target value) for quality parameter 1 in the passive and the active case.

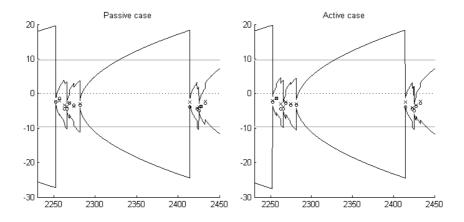


Fig. 8. Estimated values (cross), measured values (circle), uncertainties (solid black lines) and goals (grey line for largest allowable uncertainty and dotted line for target value) for quality parameter 2 in the passive and the active case.

The analysis of the validation shows in general that the predictive power of the laboratory analyzer, using the same covariance matrix Σ , has degraded substantially. Therefore, occasional laboratory measurements are needed to dynamically validate the covariance matrix. The frequency of needed laboratory measurement is, according to our analysis, much lower than current practice, as manifested in Fig. 9 which shows three measurement setups; current practice, passive case and active case. Vertical axes contain all the 14 quality measurements of laboratory analysis. With the validation data laboratory measurements were made 42 times in a three month period, which is represented on horizontal axes. In current practice every measurement was made every time, while in passive and active cases measurements were made only if the uncertainty of some estimation was too high. The

passive and the active cases are optimal in the way to maintain the accuracy high enough for all 14 of quality parameters. In the figure a black cell represents the corresponding laboratory measurement being made at the corresponding instant. Current practice costs 588 units (black cells), while passive practice would reduce costs to 170 units and active practice to 165 units.

The advantage of the active case compared to passive case appears in the dynamic validation of the a priori model (covariance matrix). As the uncertainty of the model increases too much for some quality parameter, the model has to be validated. In the passive case the 27 laboratory analyzer measurements have to be used in validation while in the active case only needed measurements could be used, as shown in Figure 10. The figure is similar in notation to Fig. 9, but vertical axes contain all 27 of laboratory analyzer measurements. In current practice measurements were made constantly, whereas in the passive and active case they are made only if needed. As a black cell represents a measurement needed, it is obvious that it is possible to reduce the number of measurements from the current practice in which costs are 1134 units, while in the passive case they are 837 units and in the active case only 334 units. In this case every quality parameter is considered separately which means that the model is validated for every quality parameter only when the uncertainty of the estimation is too high, even if the validation could have been done earlier through measurements used for validating another quality parameter. Of course, in practice, validation would have been done at that time and the number of needed measurements would have been additionally decreased. Hence, we have identified an opportunity to reduce laboratory work and focus it to where it generates most value.

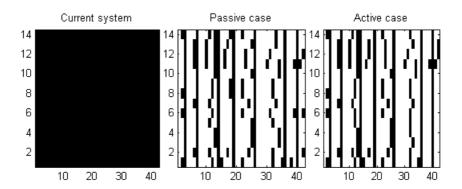


Fig. 9. Needed laboratory analysis measurements (marked with black cell) to maintain requested accuracy of the model in three month period with different measurement setups.

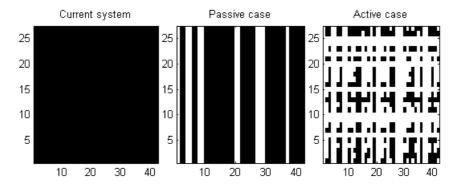


Fig. 10. Needed laboratory analyzer measurements (marked with black cell) to maintain requested accuracy of the model in three month period with different measurement setups.

It would be more effective to use fast, but often inaccurate, on-line measurements to estimate the laboratory measurements and to use laboratory analyzer measurements when validating the estimation models. The analysis with the presented framework – formal or

expert derivation of accuracy of quality information required, optimal estimation analysis of opportunities to replace labor-intensive measurements by estimates, and dynamic degradation analysis to derive frequency of measurements – however, pinpoints critical measurements and concentrates more effort on them. In most cases the analysis process itself is of high importance: it provides a shared and documented view on performance requirements for the quality measurement activities; the accuracy of information, the availability and the costs related. Knowledge about the engineered accuracy and reliability of the measurements increases the operators trust in the quality parameters, thus supporting and improving decision making.

5. CONCLUSIONS

Measurements are uncertain and estimates of real world derived from them are uncertain. Therefore operational decisions are always made under uncertainty. The value of measurement information is determined by how much the decisions can be improved based on it.

In this paper we have outlined a framework for determining the value of measurement information and designing a measurement information system/policy – what to measure and how often - based on the value generated under a defined set of scenarios. The framework in its most general form requires that we are able explicitly to define the utility function for each decision-making task and the scenarios of external effects on the system, including their frequency of occurrence. Admittedly, these are strong – in most cases unrealistic – assumptions. Therefore we noted that the result of optimal design of a measurement information system can be approximately expressed as constraint on the largest allowable uncertainty of measurement information of state, and that such constraint can be obtained from human DMs, albeit even this is not intuitive to most DMs.

We discussed the framework in a practical case of quality management at paper mills. We showed a potential for replacing some of the laborious laboratory measurements by estimates based on results from an automated laboratory analyzer. However, we also noted that for such an estimation to work over a long period of time, occasional laboratory measurements must be made to keep the estimate of covariance matrix reliable. When outlining the framework we restricted ourselves to direct value generated by the measurements, i.e. how much the measurement improves decision making by providing more accurate information about the system state. The updating of the covariance matrix is an example of indirect value generation: the measurement improves how we derive state information from other measurements. The framework will be expanded to tackle the indirect effects as well.

Decision making is a difficult task, which can be made easier by more accurate and focused measurements. The effort of making measurements can be focused when measurement of every parameter is not needed. Thus more time and energy can be used for making those few measurements and thus also calculated parameters become more accurate.

Our future research tasks include three distinct topics. One topic is to define formally the degradation of measurement information to *a priori* information, through the Ornstein-Uhlenbeck process in the case of multivariate Gaussian a priori. The second topic is to consider slow degradation of the a priori information, through a Wiener process for the multivariate Gaussian. The third topic is to solve for optimal policy via dynamic optimization, including realistic cost models for measurement combinations.

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