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FREQUENCY-RESPONSE COMPENSATION OF DISTORTING PARTS OF MEASUREMENT SYSTEMS

In this article a class of digital correction filters is presented. This class of filters has been synthesized in an optimization procedure where two criteria are considered. The first criterion is an approximation of the series interconnection of a distorting system and a compensating filter. The second is the stability of the compensating filter. Their characteristic advantage is full correction of phase distortions independent from the values of established criteria. These filters can be useful for compensation of parts of measurement systems when the discrete transfer function has the zeros on the unit circle or in its proximity. Theoretical results have been verified in numerical simulations.

Keywords: frequency-response compensation, inverse systems, regularization, noncausal systems

1. INTRODUCTION

In many measuring devices, the measured signal has often been distorted by a part of this device with an undesirable frequency response, so it is necessary to remove the effects of the measuring instrument from the data to obtain an accurate representation of the signal. In general, this kind of systems can be deemed as distorting systems. Although a distorting system is analog in most cases, the problem of correction can be considered in the discrete time domain for two following reasons:

- the best way for implementing a compensating system is to use a digital filter, because its frequency response can be freely and precisely fitted to achieve the needs,
- most measurement devices manufactured at present include a microprocessor which enables the implementation of digital signal processing algorithms as digital filtering.

In many cases, such a distorting system can be modeled as a linear time-invariant system, where the input signal is processed by the system using the convolution operation. To extract the input signal from the output signal the deconvolution operation has to be carried out. This approach is used widely in the reconstruction of spectrometric [1] and biomedical data [2] and image restoration [3] by iterative and/or regularized methods. The disadvantages of iterative methods are: the necessity of processing of signals, which have a finite length and the long computational time. Sometimes the convergence of an algorithm can be improved [4]. However, the first drawback still makes it impossible to use this kind of algorithms in the real-time reconstruction of signals.

Dynamic sensor compensation is the same class of problems. Although for many years it has been realized by means of digital signal processing [5], approaches still exist which realize the compensation using an analog system [6].

In some cases, the restoration by means of neural networks gives the good quality, which is comparable to the results obtained by the regularization method [7]. This approach is used also in equalization of digital communication channels [8]. It seems to be useful in measurand reconstruction only if the mathematical model cannot be easily obtained. When there is no information about the transfer function of a distorting system, blind deconvolution is also used. This kind of channel equalization is applied especially in digital communication systems [9].

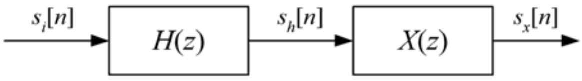


Fig. 1. Illustration of distortion compensation by an inverse system.

Assuming that the model of a distorting system is known as the transfer function of a linear, causal and time-invariant system whose coefficients are real and the digital correction filter is connected in series with the distorting system (Fig. 1), distortions in the signal $s_h[n]$ can be corrected by using a compensating system whose transfer function $X(z)$ should have the following form:

$$X(z) = \frac{1}{H(z)}. \tag{1}$$

It is necessary to mention that the problem of finding the transfer function $H(z)$ of the analog distorting system has not been considered in this article. Processing an input signal $s_i[n]$ by the series interconnection of these two systems which in this case is equivalent to the identity system, it is possible to achieve perfect correction, i.e. $s_i[n] = s_x[n]$.

It seems to be relatively simple to make such a correction for a linear time-invariant distorting system, however it is true only if $H(z)$ is the transfer function of a minimum phase system. If a distorting system is non-minimum phase then a correcting system considered as casual will not be stable. In this case, the transfer function of the distorting system can be divided into two parts as follows [10]

$$H(z) = H_{min}(z) H_{ap}(z). \tag{2}$$

$H_{min}(z)$ is designed as a minimum phase system, so it includes all poles and zeros of the transfer function $H(z)$ lying inside the unit circle and the new zeros, which are the conjugate reciprocals to the zeros of $H(z)$ lying outside the unit circle. Whereas $H_{ap}(z)$ forms an all-pass system including the zeros of $H(z)$ lying outside the unit circle and the poles which are the conjugate reciprocals to the assigned zeros. The compensating system is designed as the inversion of $H_{min}(z)$. Thus, when it is treated as casual, it will be stable. The magnitude response of $H(z)$ is exactly compensated for. Unfortunately, the phase response is not exactly compensated and is modified to the phase response of the all-pass system.

For the case when the distorting system is a non-minimum phase system, there are also approaches that use the blind deconvolution technique. The method presented in [11] allows online deconvolution of signals distorted by non-minimum phase systems with neither knowledge of this system's impulse response nor distorted signal statistics except its moments up to the fourth order.

There are domains, e.g. medicine, where compensation of signal phase distortions is a very important problem, because the useful information is not on the magnitude and phase of its Fourier transform but on its shape. Phase distortions in power measurement introduced by signal conditioning circuits have also a large influence on the measuring error [12]. Therefore, another possibility is to suppose that the exact inversion of a distorting system describes the noncausal system. Processing the signal $s_h[n]$ adequately leads to the full correction of distortions [13].

None of them assures stability of the compensating filter when the transfer function $H(z)$ of a distorting system has the zeros on the unit circle. There are solutions proposed for the case when the kernel has no spectral inverse, but they deconvolve a signal in either the frequency-domain [14] or base on finite-length sequence [15], therefore online filtering of an infinite-length signal is unfeasible. The solution proposed in this article allows to obtain stable but noncausal compensating filters. They can fully compensate the phase response of a distorting system and compensate the magnitude response on an assumed level.

2. SYNTHESIS OF THE QUASI-INVERSE FILTERS

The solution of the above-mentioned problem can be carried out by compromise, using a compensating filter that is stable and in series connection with the distorting system, which would form a system in all means similar to the identity system. It has been decided that searching for the solution will be treated as an optimization problem, which will consider the two following criteria:
- stability of the compensating filter,
- approximation of the identity system by the series interconnection of the distorting system and the compensating filter.

Minimization of indicators of these two criteria makes it possible to determine the optimal solution.

The mathematical form of this problem has been solved algebraically assuming that each real-valued signal is represented by the column vector in the multidimensional space, i.e. any signal $a[n]$ has an equivalent vector

$$\mathbf{a} = [\ldots, a_{-2}, a_{-1}, a_0, a_1, a_2, \ldots]^T , \tag{3}$$

where an corresponds to the n-th sample of the signal $a[n]$. It was assumed that the indexes of the vector coefficients can be negative, because, in general, these vectors can represent noncausal signals. Therefore, the vectors must always have odd numbers of the coefficients to unambiguously determine the sample with index 0. It can be always achieved by adding the zeros to a vector to position the sample with the index 0 into the middle of a vector.

The indexes of the aforementioned criteria for the optimization procedure have been described mathematically by the inner product. The following two indexes have been defined:
- The stability index

$$c(\mathbf{x}) = \langle \mathbf{x}, \mathbf{x} \rangle = \frac{1}{2\pi\,\mathrm{j}} \oint_{|z|=R} |X(z)|^2 z^{-1} dz . \tag{4}$$

  where $R > \max_k |p_k^X|$ and p_k^X is k-th pole of the transfer function $X(z)$. Its value describes the energy of the searched filter impulse response. If its value is finite, then the obtained filter certainly will be stable asymptotically. The lower the value of this index, the higher the stability of the searched filter (the poles of this filter lie further away from the unit circle). The definition of this index on the basis of the impulse response energy follows from the fact that it can be written by using the inner product.
- The approximation index

$$f(\mathbf{x}) = \langle \mathbf{H}\mathbf{x} - \boldsymbol{\delta}, \mathbf{H}\mathbf{x} - \boldsymbol{\delta} \rangle = \frac{1}{2\pi\,\mathrm{j}} \oint_{|z|=R} |H(z)X(z) - 1|^2 z^{-1} dz , \tag{5}$$

where $\mathbf{H}$ is the Toeplitz matrix which is built out of the impulse response $h[n]$ of the distorting system (the columns of this matrix are the successively delayed replicas of $h[n]$), $\mathbf{x}$ is the impulse response $x[n]$ of a compensating system and δ is the discrete unit impulse. In this case $R > \max\left\{\max_k \left|p_k^X\right|, \max_k \left|p_k^H\right|\right\}$ where p_k^H is k-th pole of the transfer function $H(z)$.The value of this index describes the level of the identity system approximation by the series interconnection of the distorting system and the searched for compensating system. The lower the value of this index, the better the approximation of the identity system.

Equivalent dependences in (4) and (5) described in the Z transform domain follow from the Parseval relation.

2.1. Optimization problems

In a situation when the discrete transfer function of a distorting system has the zeros on the unit circle, the search for the stable inversion can be carried out by minimization of the first presented index with the assumed constant value of the other one. Therefore, the proposed procedure can be realized in the following two ways.

- A compensating filter with the minimum value of the stability index is searched, i.e.

$$f_1(\mathbf{x}) = \langle \mathbf{x}, \mathbf{x} \rangle \to \min . \tag{6}$$

Additionally, the constraint is formed

$$c_1(\mathbf{x}) = \langle \mathbf{Hx} - \boldsymbol{\delta}, \mathbf{Hx} - \boldsymbol{\delta} \rangle = q_1 , \tag{7}$$

which means that the approximation index has to be equal to q_1.

- A compensating filter which, in connection with the distorting system, forms the best approximation of the identity system, is searched for, i.e.

$$f_2(\mathbf{x}) = \langle \mathbf{Hx} - \boldsymbol{\delta}, \mathbf{Hx} - \boldsymbol{\delta} \rangle \to \min . \tag{8}$$

The second index forms the constraint

$$c_2(\mathbf{x}) = \langle \mathbf{x}, \mathbf{x} \rangle = q_2 , \tag{9}$$

which means that the stability index has to be equal to q_2.

The presented assumptions will be called the first and second optimization problem, respectively. It seems that assuming the approximation index is more intuitive than the stability index. Nevertheless, both optimization problems have been solved to show that they lead to equivalent solutions. In the next section, the procedure for obtaining the solution of the first optimization problem is shown. The second can be solved identically.

2.2. Solutions of the optimization problems

In order to group the requirements (6) and (7) in the first optimization problem, the Lagrange functional is determined

$$L_1(\mathbf{x}, \lambda) = f_1(\mathbf{x}) + \lambda c_1(\mathbf{x}) = \langle \mathbf{x}, \mathbf{x} \rangle + \lambda\left(\langle \mathbf{Hx} - \boldsymbol{\delta}, \mathbf{Hx} - \boldsymbol{\delta} \rangle - q_1\right), \tag{10}$$

where $\lambda \in R^{+}$ is the Lagrange multiplier. For the established value of λ the functional (10) can attain the minimum $\mathbf{x}_{\lambda}$ when for any variation Δ_x of the vector $\mathbf{x}_{\lambda}$ the following inequality is always true:

$$L_1(\mathbf{x}_{\lambda} + \mathbf{\Delta}_x, \lambda) - L_1(\mathbf{x}_{\lambda}, \lambda) > 0 . \tag{11}$$

Applying the axioms and the properties of the inner product, the inequality (11) takes the form:

$$2\left\langle \mathbf{x}_{\lambda} + \lambda \mathbf{H}^{*}\mathbf{H}\mathbf{x}_{\lambda} - \lambda \mathbf{H}^{*}\boldsymbol{\delta}, \mathbf{\Delta}_x \right\rangle + \lambda \left\| \mathbf{H}\mathbf{\Delta}_x \right\|^2 + \left\| \mathbf{\Delta}_x \right\|^2 > 0 , \tag{12}$$

where $\mathbf{H}^{*}$ denotes generally the conjugate transpose of matrix $\mathbf{H}$. This operation for linear time-invariant systems with the real-valued samples of the impulse response is simply the transposition of the matrix $\mathbf{H}$.

The condition (11) is satisfied for any variation of Δ_x, if and only if the first component of the inner product in (12) equals zero, i.e.:

$$\mathbf{x}_{\lambda} + \lambda \mathbf{H}^{*}\mathbf{H}\mathbf{x}_{\lambda} - \lambda \mathbf{H}^{*}\boldsymbol{\delta} = \mathbf{0} . \tag{13}$$

It is a necessary and sufficient condition for the minimum of the functional (10) to exist. Rearranging the Eq. (13) with respect to x_{λ}, the following equation is obtained:

$$\mathbf{x}_{\lambda} = \left(\mathbf{1} + \lambda \mathbf{H}^{*}\mathbf{H}\right)^{-1} \lambda \mathbf{H}^{*}\boldsymbol{\delta} = \lambda \mathbf{H}^{*}\left(\mathbf{1} + \lambda \mathbf{H}^{*}\mathbf{H}\right)^{-1} \boldsymbol{\delta} . \tag{14}$$

The formula (14) makes it possible to directly calculate the impulse response of the compensating filter. The multiplication of convolution operators is commutative, so (14) can be shown as the following fraction:

$$\mathbf{x}_{\lambda} = \frac{\lambda \mathbf{H}^{*}}{\mathbf{1} + \lambda \mathbf{H}^{*}\mathbf{H}} \boldsymbol{\delta} . \tag{15}$$

This relationship defines the set of filters called the λ-family of the quasi-inverse filters.

Identically the following relationship has been obtained:

$$\mathbf{x}_{\lambda} = \frac{\mathbf{H}^{*}}{\lambda \mathbf{1} + \mathbf{H}^{*}\mathbf{H}} \boldsymbol{\delta} , \tag{16}$$

which represents the solution of the second optimization problem.

Taking (15), (16) and the definitions of the indexes of the approximation and stability into consideration for each of the problems, the approximation $A(\lambda)$ and stability $S(\lambda)$ functions have been determined.

$$A_1(\lambda) = \left\langle \frac{\lambda \mathbf{H}^{*}}{\mathbf{1} + \lambda \mathbf{H}^{*}\mathbf{H}} \boldsymbol{\delta} - \boldsymbol{\delta}, \frac{\lambda \mathbf{H}^{*}}{\mathbf{1} + \lambda \mathbf{H}^{*}\mathbf{H}} \boldsymbol{\delta} - \boldsymbol{\delta} \right\rangle , \tag{17}$$

$$S_1(\lambda) = \left\langle \frac{\lambda \mathbf{H}^{*}}{\mathbf{1} + \lambda \mathbf{H}^{*}\mathbf{H}} \boldsymbol{\delta}, \frac{\lambda \mathbf{H}^{*}}{\mathbf{1} + \lambda \mathbf{H}^{*}\mathbf{H}} \boldsymbol{\delta} \right\rangle , \tag{18}$$

$$A_2(\lambda) = \left\langle \frac{\mathbf{H}^*}{\lambda\mathbf{1} + \mathbf{H}^*\mathbf{H}}\boldsymbol{\delta} - \boldsymbol{\delta}, \frac{\mathbf{H}^*}{\lambda\mathbf{1} + \mathbf{H}^*\mathbf{H}}\boldsymbol{\delta} - \boldsymbol{\delta} \right\rangle, \tag{19}$$

$$S_2(\lambda) = \left\langle \frac{\mathbf{H}^*}{\lambda\mathbf{1} + \mathbf{H}^*\mathbf{H}}\boldsymbol{\delta}, \frac{\mathbf{H}^*}{\lambda\mathbf{1} + \mathbf{H}^*\mathbf{H}}\boldsymbol{\delta} \right\rangle. \tag{20}$$

An analysis of the graph of these functions can show the variations of the approximation and the stability index with respect to the variations of λ.

According to the constraints (7) and (9), the approximation index in the first optimization problem or the stability index in the second optimization problem has to be equal to the assumed value. Therefore, in order to obtain the optimal solution for the established assumption, the value of λ has to be determined. In the first optimization problem, it is equivalent to solve the equation:

$$A_1(\lambda) = q_1. \tag{21}$$

In the second optimization problem, to determine the value of λ the following equation has to be solved:

$$S_2(\lambda) = q_2. \tag{22}$$

In both cases, the solution can be obtained using Newton's method [16].

3. TRANSFER FUNCTION OF THE QUASI-INVERSE FILTERS

Using the properties of the Z transform, the solution (15) can be easily transformed to the transfer function $X_\lambda(z)$ of the quasi-inverse filter. For the first optimization problem, it has the following form:

$$X_\lambda(z) = \frac{\lambda H(z^{-1})}{1 + \lambda H(z^{-1})H(z)}. \tag{23}$$

Dividing the nominator and the denominator of (23) by λ, we obtain:

$$X_\lambda(z) = \frac{H(z^{-1})}{\frac{1}{\lambda} + H(z^{-1})H(z)}. \tag{24}$$

In the same way, we can obtain the transfer function of the filter in the second optimization problem:

$$X_\lambda(z) = \frac{H(z^{-1})}{\lambda + H(z^{-1})H(z)}. \tag{25}$$

The Equations (24) and (25) are similar if:

$$\lambda \leftrightarrow \frac{1}{\lambda}. \tag{26}$$

Therefore, all corollaries related to some variations of λ which will be shown for the class of the filters described by (23) will be also true for the filters described by (25), but for the inverse variations of λ. It can be noticed too by comparison of the plots of the approximation and the stability functions, which will be shown in section 5.

It can be observed that the transfer functions (24) and (25) are similar to the transfer function of the linear equalizer obtained on the basis of the MSE criterion [17]. The difference is in the meaning of the factor λ, which in this kind of equalizer describes the noise spectral density. However, in these two equations this factor controls either the stability of the correction filter or the approximation level of the overall system depending of the approximation problem.

4. FREQUENCY-RESPONSE OF THE QUASI-INVERSE FILTERS

Applying the substitution $z = e^{jw}$ in (30) and determining λ based on the assumed q_1 value, the frequency response of the quasi-inverse filter obtained for the first optimization problem can be determined.

$$X_\lambda\left(e^{j\omega}\right) = \frac{H\left(e^{-j\omega}\right)}{\frac{1}{\lambda} + H\left(e^{-j\omega}\right)H\left(e^{j\omega}\right)} = \frac{H\left(e^{-j\omega}\right)}{\frac{1}{\lambda} + \left|H\left(e^{j\omega}\right)\right|^2}, \tag{27}$$

where $\omega = 2\pi f/f_p$. Rewriting the nominator of (27) as the module and argument:

$$H\left(e^{-j\omega}\right) = \left|H\left(e^{j\omega}\right)\right| e^{-j\arg\left[H\left(e^{j\omega}\right)\right]}, \tag{28}$$

the magnitude response can be determined:

$$\left|X_\lambda\left(e^{j\omega}\right)\right| = \frac{\left|H\left(e^{j\omega}\right)\right|}{\frac{1}{\lambda} + \left|H\left(e^{j\omega}\right)\right|^2} \tag{29}$$

and the phase response:

$$\arg\left[X_\lambda\left(e^{j\omega}\right)\right] = -\arg\left[H\left(e^{j\omega}\right)\right], \tag{30}$$

of the quasi-inverse filter. It can be observed that the multiplier λ influences only the magnitude response of the quasi-inverse filter. The phase response (30) is independent of λ and is the exact inversion of the phase response of the distorting system. Owing to this property, quasi-inverse filters can correct exactly the phase distortions regardless of the value of λ. Of course, in practice exact correction is achieved only when we have an exact model of the distorting system e.g. the transfer function or the impulse response, because the quasi-inverse filter is designed on the basis of this model.

5. APPROXIMATION AND STABILITY FUNCTIONS

Under the Parseval relation, the definitions (17)-(20) of the approximation and stability functions can be written in the Z transform domain. Knowledge of the transfer functions of the quasi-inverse filters allows to analytically determine the plots of the approximation and stability functions for both optimization problems (Fig. 2 and 3).

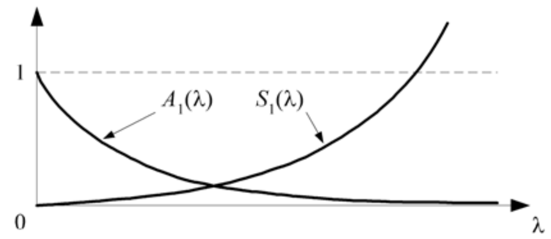


Fig. 2. Plots of the approximation $A_1(\lambda)$ and stability $S_1(\lambda)$ functions.

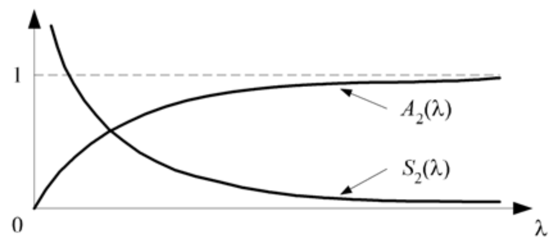


Fig. 3. Plots of the approximation $A_2(\lambda)$ and stability $S_2(\lambda)$ functions.

It can be observed that for the specific optimization problem, graphs of both functions have the opposite trends i.e. the better the approximation, the worse the stability and vice versa. However, owing to monotony of these functions, the Eqs. (21) and (22) have always one and only one solution.

6. POLE-LOCATIONS FOR THE QUASI-INVERSE FILTERS

Let us suppose that a causal distorting system has been described by the polynomial transfer function:

$$H(z)=\sum_{k=0}^{N} h_k z^{-k} \;. \tag{31}$$

This assumption does not influence the generality of the presented considerations, because any stable discrete system can be described, with sufficient precision, by a polynomial transfer function $H(z)$. Substituting (31) into (25) leads to:

$$X_\lambda(z)=\frac{\sum_{k=0}^{N} h_k z^{k}}{\lambda+\sum_{k=0}^{N} h_k z^{k}\sum_{k=0}^{N} h_k z^{-k}} \;. \tag{32}$$

Equality of the coefficients in the polynomials in (32), presented shortly as the sums, makes that after the multiplication of these polynomials in the denominator, a symmetrical polynomial is obtained. It means that the coefficients of this polynomial with the same indexes as for its absolute value, have the same values. Therefore, (32) can be rewritten to the form:

$$X_\lambda(z)=\frac{\sum_{k=0}^{N} h_k z^k}{\sum_{k=-N}^{N} c_k z^k}, \tag{33}$$

where

$$c_k=\begin{cases}\lambda+\sum_{i=0}^{N} h_i^2 & \text{dla} \quad k=0\\ \sum_{i=0}^{N-k} h_i h_{k+i} & \text{dla} \quad |k|=1,2,\ldots,N\end{cases}. \tag{34}$$

Symmetry of the polynomial in the denominator of the transfer function (33) means that the complex poles appear by four, i.e. if p is the pole of the transmit function (33), then there exists the complex-conjugate pole p^* and two poles p^{-1} and $(p^*)^{-1}$ conjugate reciprocally to the poles p^* and p, respectively. It is shown in Fig. 4. This pattern of the pole locations for the quasi-inverse filters appears also if the transfer function of the distorting system is rational. Therefore, in order to determine all poles for the quasi-inverse filter, it is sufficient to know the locations of the poles which lie above the real axis and inside the unit circle (striped area in Fig. 4).

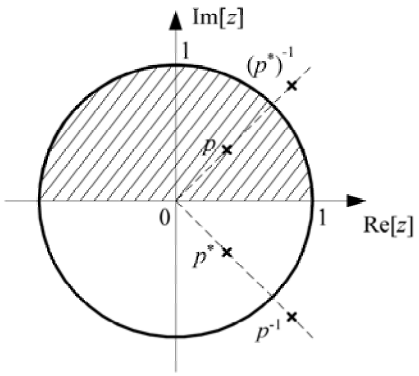


Fig. 4. Pattern of pole locations for the quasi-inverse filters.

Regarding the poles lying outside the unit circle, the quasi-inverse filters have to be treated as noncausal systems. Otherwise, they are unstable. To properly process a signal, the structure of the quasi-inverse filter has to be divided into two parts. One of them should group all poles lying inside the unit circle, whereas the second should group all poles lying outside the unit circle. Two stable systems are obtained because of that division: causal and anticausal, respectively. In addition, the anticausal part should process the signal in reversed order [18]. If the input signal has a finite number of the samples, this limitation does not make any problems. However, if the processed signal has an infinite number of samples, the realization of the filtration is more difficult, but possible. In order to realize the real-time filtering of either a very long or an infinite input signal by the anticasual part, the block convolution can be used. This procedure can be achieved by the overlap-add or overlap-save method. A very useful implementation of the overlap-add method is presented in [19] and evolved in [20, 21, 22], where two LIFO stacks are used for the time-reversed convolution.

7. SIMULATION EXAMPLE

In this example, the hypothetical distorting system has been described by the rational transfer function. Its frequency response is shown in Fig. 5, where $F = f/f_s$. The poles of this system (Fig. 6) lie on the unit circle or in its proximity.

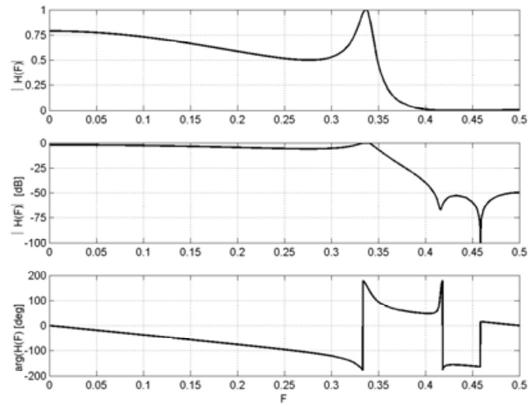


Fig. 5. Frequency response for the hypothetical distorting system.

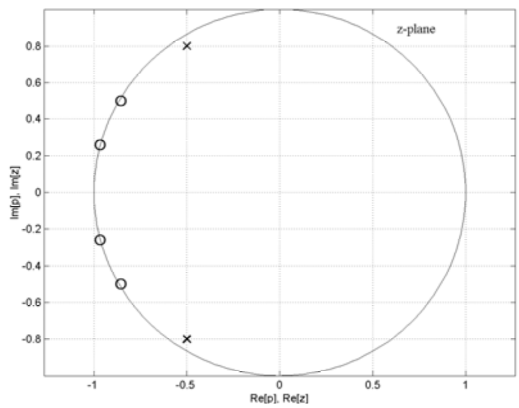
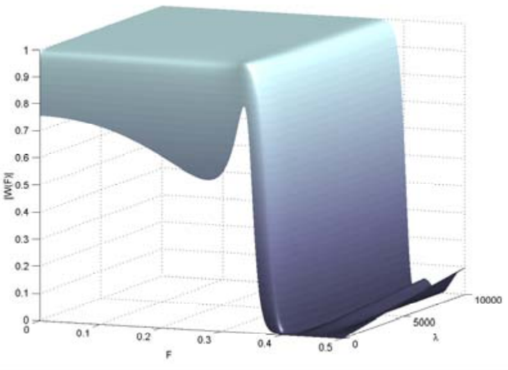


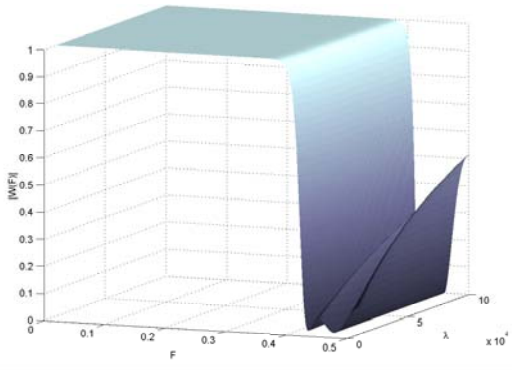
Fig. 6. Pole-zero plot for the hypothetical distorting system.

Because of the pole locations, the compensating system, designed as the exact inversion and deemed causal, is unstable. To obtain a stable compensating system, the quasi-inverse filter can be used. Because the assumption of the approximation index is more intuitive, the solution for the first optimization problem is used in this example.

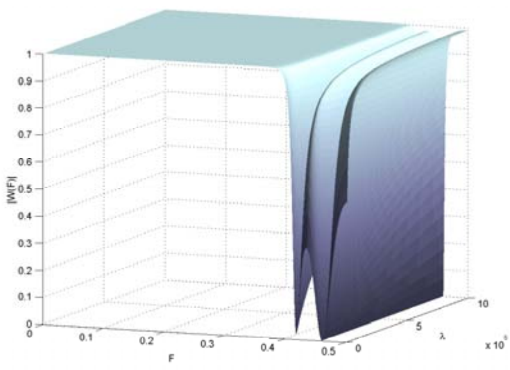
In order to show the changes of the magnitude response $|W(F)|$ of the overall system, resulting from the cascade connection of the quasi-inverse compensating filter and a distorting system, depending on the various values of λ, the 3D plot is used for its presentation (Fig. 7).



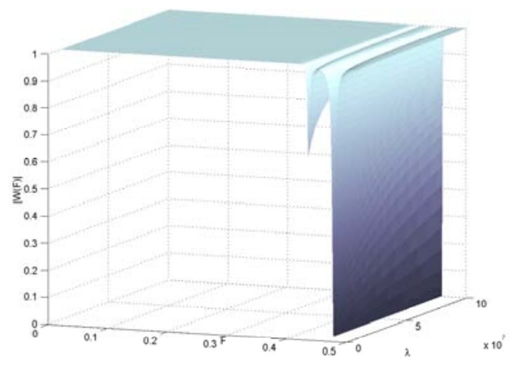
a) $\lambda = 5..10^4$



b) $\lambda = 10^4..10^5$



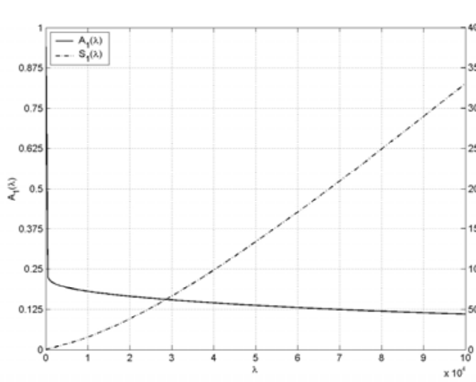
c) $\lambda = 10^5..10^7$



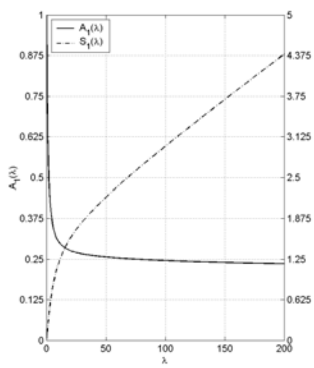
d) $\lambda = 10^7..10^8$

Fig. 7. Magnitude responses for the overall system for various values of λ.

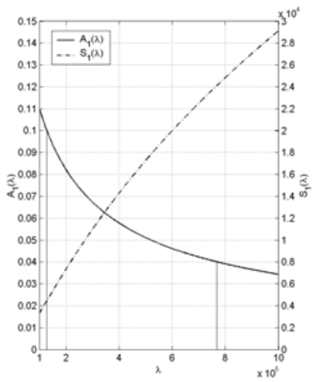
Selection of the value of q_1 is always a compromise between the acceptable fall of the approximation level of the identity system and the desirable stability of the quasi-inverse compensating filter. The values of the indexes, which characterize these properties, can be read from the graphs of the approximation and stability functions shown in Fig. 8. Figure 8 b shows the same plots as Fig. 8 a, but for shorter changes of λ to show the shape of the presented functions for small values of λ.



a) $\lambda = 0..10^5$



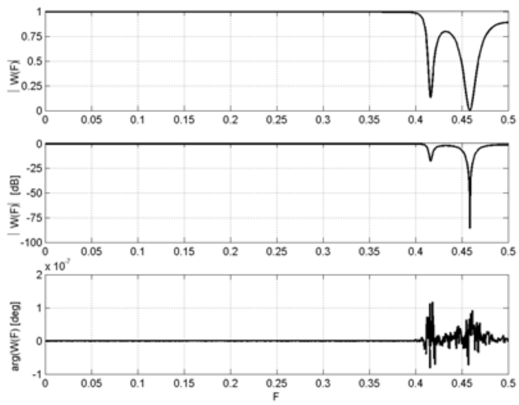
b) $\lambda = 0..500$



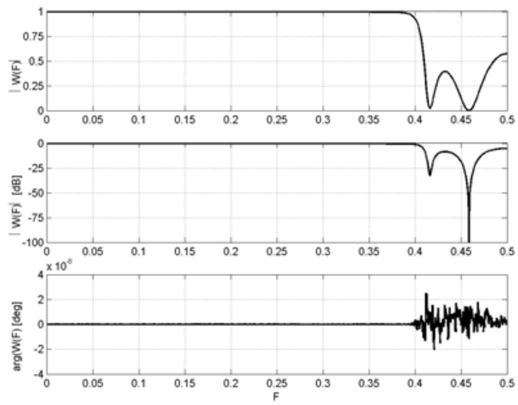
c) $\lambda = 10^5..10^6$

Fig. 8. Approximation $A_1(\lambda)$ stability $S_1(\lambda)$ functions for the quasi-inverse filter in this example.

Frequency responses of the resultant overall system and the pole location of the quasi-inverse filter for two values of q_1 are shown in Fig. 9 and 10, respectively. Corresponding values of λ are showed in the captions of these figures and in Fig. 8 c. As can be observed and as mentioned before, the higher the value of the approximation index, the better the stability of the compensating system and the worse the approximation of the resultant system.

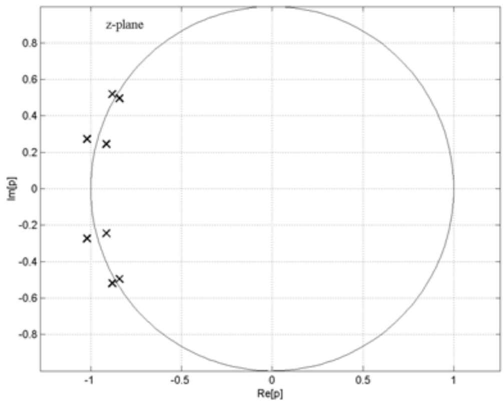


a) $q_1 = 0.04$; $\lambda = 787336$

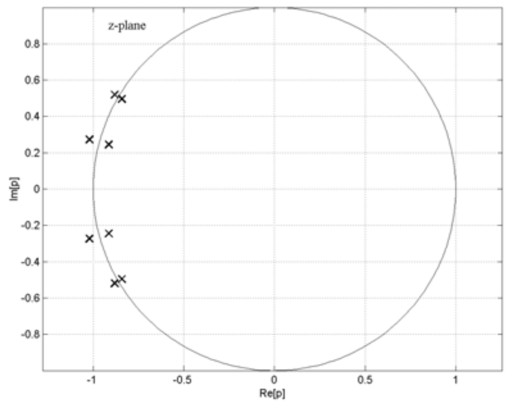


b) $q_1 = 0.1$; $\lambda = 128196$

Fig. 9. Frequency responses for the overall system for two values of q_1.



a) $q_1 = 0{,}04$; $\lambda = 787336$



b) $q_1 = 0{,}1$; $\lambda = 128196$

Fig. 10. Pole plot for the quasi-inverse filter for two values of q_1.

8. CONCLUSION

In this article, the method of stable and noncausal quasi-inverse filters has been shown. They can be used for compensation of distortions introduced to the measured signal, when the system function $H(z)$ or the impulse response $h[n]$ of this distorting system is known. Especially, they can be useful in the compensation of the frequency response of parts of measurement channels, when their discrete transfer function has the zeros on the unit circle or in its proximity.

The optimization approach used in the process of synthesis allows to choose the approximation level of the overall system resulting from the cascade connection of this filter and a distorting system, or the stability of this quasi-inverse filter.

Based on the algebraic solutions, the transfer function and the characteristic pattern of poles locus of this class of filters has been shown. The undoubted feature of these filters is the exact inversion of the phase response with relation to the phase response of the distorting system. This allows for full correction of phase distortions.

Even though this family of filters belongs to the class of noncausal systems, as a result of proper processing of input signals by the sectioned convolution method it is possible to process either very long or infinite signals in real time.

It would be interesting to include into the synthesis of quasi-inverse filters the noise influence on the distorted signal, which is planned in future work.

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KOREKCJA CHARAKTERYSTYK CZĘSTOTLIWOŚCIOWYCH ZNIEKSZTAŁCAJĄCYCH CZĘŚCI TORU POMIAROWEGO

Streszczenie

W artykule zaprezentowano klasę cyfrowych filtrów korekcyjnych zsyntetyzowaną w oparciu o procedurę optymalizacyjną uwzględniająca kryteria aproksymacji układu wypadkowego powstałego przez szeregowe dołączenie do układu zniekształcającego tegoż filtru korekcyjnego i stabilności poszukiwanego filtru korekcyjnego. Ich szczególną zaletą jest pełna kompensacja zniekształceń fazowych niezależnie od wartości wskaźników przyjętych kryteriów optymalizacyjnych. Zastosowanie niniejszych filtrów korekcyjnych jest ukierunkowane na przypadki, kiedy dyskretna transmitancja układu zniekształcającego posiada zera na okręgu jednostkowym lub w jego bliskiej odległości.