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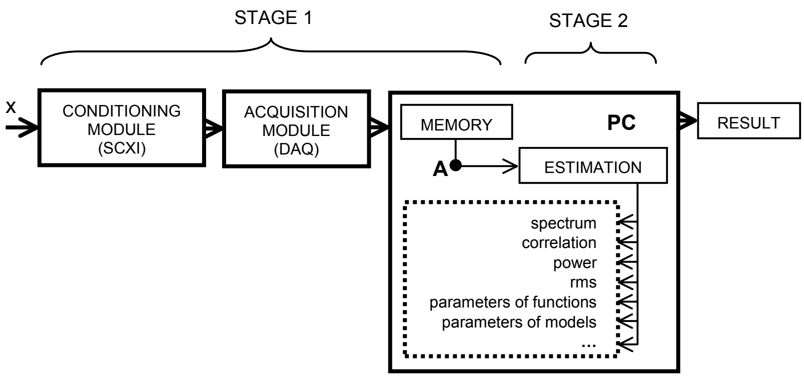
THE IMPACT OF THE RANDOMIZATION OF THE QUANTIZATION ERROR ON THE ACCURACY OF MEASURING SYSTEMS APPLYING A DIGITAL MEASURING ALGORITHM

In measuring systems applying a digital measuring algorithm, the measurement process is divided into two stages: Stage 1 consists in the acquisition and storage of the system's input data, based on which the value of the measurand is estimated in stage 2. The uncertainty of the stored data is translated into the uncertainty of the result of the estimation, i.e. of the result of the measurement. When analog-digital conversion with a dither signal is applied in the data acquisition system (DAQ), the quantization error is modified and the uncertainty is reduced. The present paper discusses the mechanism of the modification of the quantization error and describes the characteristics of the uncertainty of a DAQ system applying a-d conversion with a dither signal.

Keywords: a-d conversion, dither, quantization error, uncertainty

1. INTRODUCTION

In measuring systems with a digital measuring algorithm, the data are processed through a combination of the system's and the software's operations. The system accounts for stage 1 of the data processing, i.e. the acquisition of data at the input of the system, which data are subsequently discretized and stored. The software, in turn, accounts for stage 2, i.e. the estimation. The purpose of stage 2 is to provide the result of the measurement by using methods of digital data processing, or algorithms applied to the input data stored in stage 1. Fig. 1 presents the overall organization of a measuring system with a digital measuring algorithm.



SCXI - Signal Conditioning eXtensions for Instrumentation
DAQ - Data Acquisition

Fig. 1. The overall organization of a measuring system with a digital measuring algorithm.

Measuring systems with a digital measuring algorithm may be described as follows:

- the a-d conversion takes place extremely close to the location of the acquisition of the data which provide information on the values of the measurand,

- the system's operations on the measurement signal are minimized, and the main measurement processing is done by means of software.

In order to determine the uncertainty of the result of the measurement, the uncertainty of the result of the estimation is assessed. The latter uncertainty is primarily due to the uncertainty of the stored data, which in turn provides the basis for establishing the uncertainty of the result of the measurement. The uncertainty of the stored data may be assessed by means of establishing the value of the uncertainty of the results of the conditioning and the acquisition. Fig. 2 illustrates the relationships between all of the above uncertainties.

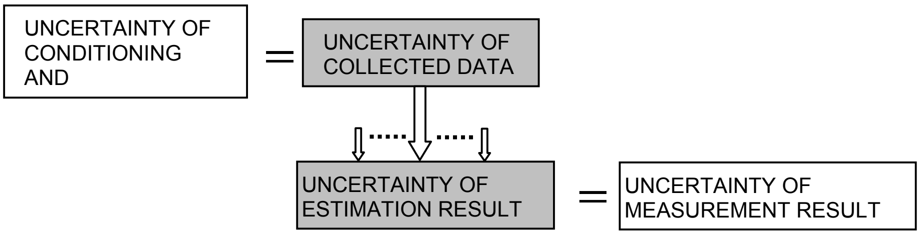


Fig. 2. The relationships between the uncertainties of a measuring system with a digital measuring algorithm.

The combined standard uncertainty u_c at point A (Fig. 2) is defined as follows:

$$u_c = \sqrt{u_{cSCXI}^2 + u_{cDAQ}^2}\ , \tag{1}$$

u_{cSCXI} - the standard uncertainty of the data at the output of an SCXI module,
u_{cDAQ} - the standard uncertainty of the data at the output of a DAQ system.

This dependence is received applying the methodology of evaluating of uncertainty described in [1] (in agreement with principles prescribed in [2]). At that, components of total error (SCXI+DAQ), being products of elementary errors have been omitted.

The present paper addresses itself to the issues of the modification of a-d conversion and its impact on the uncertainty u_{cDAQ}. Accordingly, only this type of uncertainty is discussed henceforth.

2. THE CHARACTERISTICS OF THE ACCURACY OF A DAQ SYSTEM

Five types of errors determine the accuracy of a DAQ system [5]: the gain error Δ_G, the offset error Δ_{OFF}, the nonlinearity error Δ_{NL}, the noise error Δ_n, and the quantization error Δ_q. The accuracy parameters are specified for a calibrated DAQ system. When a system works in temperatures outside the rated range of 15° - 35° C, the error due to the temperature drift must also be taken into consideration. In the rated temperature range, the error e_i of a single value of stored data x_i, which is due to the errors of the DAQ system, amounts to:

$$e_i = \Delta_{Gi} x_i + \Delta_{OFF} + \Delta_{NLi} + \Delta_{ni} + \Delta_{qi}\ . \tag{2}$$

While the values of the components of the error e_i are not known, we know the limits of the ranges within which such components are contained:

$$|\Delta_G| \le M_G\ ,\ |\Delta_{OFF}| \le M_{OFF}\ ,\ |\Delta_{NL}| \le M_{NL}\ ,\ |\Delta_q| \le q/2\ , \tag{3}$$

M - the boundary value of the error Δ,
q - the resolution of the A/D converter.

For noise n, we also know its distribution (which is Gaussian) and standard deviation σ_n.

The uncertainty of the result of the conversion is determined based on the following assumptions:

- the various errors and their distributions are independent of one another,
- the average values of the errors equal zero,
- the gain error, the offset error, the nonlinearity error and the quantization error have uniform distributions.

The values of the standard uncertainty amount to:

$$u_G = \frac{M_G}{\sqrt{3}} \quad , \quad u_{OFF} = \frac{M_{OFF}}{\sqrt{3}} \quad , \quad u_{NL} = \frac{M_{NL}}{\sqrt{3}} \quad , \quad u_q = \frac{q/2}{\sqrt{3}} \quad , \quad u_n = \sigma_n . \tag{4}$$

The combined standard uncertainty due to the errors of the DAQ system is established by means of the formula [2]:

$$u_{cDAQ\_1} = \sqrt{u_G^2 x^2 + u_{OFF}^2 + u_{NL}^2 + u_n^2 + u_q^2} . \tag{5}$$

In order to modify the characteristics and parameters of quantization, a-d conversion with a dither signal is applied. We now proceed to discuss the effect of this modification on the uncertainty u_{cDAQ}.

3. A-D CONVERSION WITH A DITHER SIGNAL: THE RANDOMIZATION OF THE QUANTIZATION ERROR

The dither signal d is an extra signal added to the signal undergoing conversion x prior to its a-d conversion and converted together with it. The purpose of the dither signal is to modify the characteristics of the operations of quantization and to enable their examination and control [4]. The variety of a-d conversion with a dither signal applied in measuring systems is the one shown in Fig. 3, where the results of the a-d conversion are averaged (if multiple measurements of the same quantity are possible) or filtrated (in which case an FIR digital filter is applied). The dither signal is supplied by a software-controlled generator.

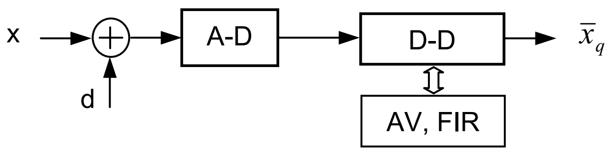


Fig. 3. A-D conversion with a dither signal.

The dither signal is typically white Gaussian noise or noise with a uniform probability density function. The designs of DAQ systems feature such dither generators.

Due to the application of a dither signal, the quantization error is randomized and turned into additive noise, subsequently eliminated by means of the averaging or filtration of the results of the a-d conversion. If the dither signal has been selected optimally, then both the randomization and the elimination are complete. Conversely, if the dither signal has not been selected optimally, then both the randomization and the elimination are partial only. The standard procedure is to average 100 values of the quantity acquired at the input of the system, although in certain circumstances 1000 of these values are averaged.

The a-d conversion with dither signal (Fig. 3) is equivalent to the a-d conversion with modified error of the operation of quantization. The error of the operation of quantization g and of quantization with dither and average $\overline{g}$ is defined respectively as:

$$g(x) = Q(x) - x \ , \qquad \overline{g}(x) = \overline{Q}(x) - x \, , \qquad (6)$$

Q- the input-output characteristic of an ideal A/D converter.

Fig. 4 illustrates the impact of the application of a-d conversion with a dither signal on the error of the operation of quantization.

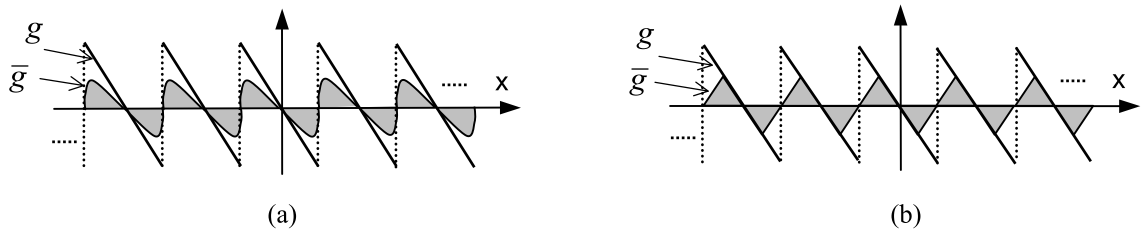


Fig. 4. The errors of the operations of quantization q and of quantization with a dither signal and averaging $\overline{q}$: a) normal dither signal, b) uniform dither signal.

4. THE CHARACTERISTICS OF THE ACCURACY OF A DAQ SYSTEM APPLYING A-D CONVERSION WITH A DITHER SIGNAL

The application of a dither signal results in a partial randomization of the quantization error. In such a case, the quantization error of a-d conversion (Fig. 3) consists of the non-randomized component $\overline{q}$ and the randomized component $q - \overline{q}$. The randomized component is added to the noise of the DAQ system and is reduced by means of averaging. The non-randomized component persists, and constitutes the error of the operations of quantization characteristic of a-d conversion with a dither signal. In this situation, the combined standard uncertainty caused by the errors of the DAQ system is established as follows:

$$u_{cDAQ\_2} = \sqrt{u_G^2 x^2 + u_{OFF}^2 + u_{NL}^2 + u_{\overline{q}}^2 + \frac{u_n^2 + \left(u_q^2 - u_{\overline{q}}^2\right)}{m}} \, , \qquad (7)$$

$u_{\overline{q}}^2$ - the variance due to the non-randomized component of the quantization error $\overline{q}$,

$u_q^2 - u_{\overline{q}}^2$ - the variance due to the randomized component of the quantization error,

m - the number of the averaged results of the acquisition (of the values of a certain quantity).

The standard uncertainties $u_{\overline{q}}$ for a Gaussian dither signal of the variance σ^2 and for a dither signal with a uniform probability density function and with values from the range of [–w/2, w/2] are expressed respectively by the formulae [3]:

$$u_{\overline{q}} = \frac{q}{\sqrt{2}\pi} \sqrt{\sum_{k=1}^{\infty} \frac{1}{k^2} \exp\left(-4\pi^2 \frac{\sigma^2}{q^2} k^2\right)}, \quad u_{\overline{q}} = \frac{q^2}{\sqrt{2}\pi^2 w} \sqrt{\sum_{k=1}^{\infty} \frac{1}{k^4} \sin^2\left(\pi \frac{w}{q} k\right)} \, . \qquad (8)$$

A Gaussian dither signal has been optimally selected if the ratio of σ/q equals 0.5; a dither signal with a uniform probability density function has been optimally selected if the ratio of w/q is a natural number (usually w/q = 1). If the dither signal is optimal, then $\overline{q} = 0$ and $u_{\overline{q}} = 0$, and therefore the entire quantization error is randomized and eliminated. The combined standard uncertainty (7) is reduced to the formula:

$$u_{cDAQ\_3} = \sqrt{u_G^2 x^2 + u_{OFF}^2 + u_{NL}^2 + \frac{u_n^2 + u_q^2}{m}} \, . \qquad (9)$$

The following relationships are effective between the uncertainties (5), (7) and (9):

$$u_{cDAQ\_1} > u_{cDAQ\_2} > u_{cDAQ\_3} \, , \qquad (10)$$

because

$$u_n^2 + u_q^2 > u_{\overline{q}}^2 + \frac{u_n^2 + \left(u_q^2 - u_{\overline{q}}^2\right)}{m} > \frac{u_n^2 + u_q^2}{m} \, . \qquad (11)$$

It is inferred from these relationships that the application of a-d conversion with a dither signal and averaging diminishes the combined standard uncertainty of a DAQ system. The accuracy is always improved, even when the dither signal cannot be selected optimally $\left(u_{\overline{q}}^2 \neq 0\right)$.

5. CONCLUSIONS

A-D conversion with a dither signal has so far been applied in measuring systems in order to examine and control the various parameters of the operations of quantization. The present paper demonstrates that the application of such conversion also results in diminishing the uncertainty of the result of the measurement in a quantifiable manner. The accuracy is always improved, even when the dither signal cannot be selected optimally $\left(u_{\overline{q}}^2 \neq 0\right)$.

The uncertainty of the result of the measurement diminishes due to the diminution of the uncertainty of the stored data; the latter effect is produced by randomizing the quantization error, which subsequently may be eliminated by means of averaging or filtration. The extent to which the quantization error may be randomized and eliminated depends on generating a Gaussian dither signal with a proper standard deviation (relative to the resolution of the applied A/D converter) or a dither signal with a uniform probability density function and with values from the proper range (relative to the resolution of the applied A/D converter).

It is very easy to implement a-d conversion with a dither signal in measuring systems applying a digital measuring algorithm, as the DAQ systems incorporated in such measuring systems are always provided with software-controlled generators of dither signals.

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WPŁYW RANDOMIZACJI BŁĘDU KWANTOWANIA NA DOKŁADNOŚĆ SYSTEMÓW POMIAROWYCH Z CYFROWYM ALGORYTMEM POMIARU

Streszczenie

W systemach pomiarowych z cyfrowym algorytmem pomiaru proces pomiarowy przebiega w dwóch etapach. W pierwszym etapie zbierane i zapamiętywane są dane wejściowe systemu. Na ich podstawie w drugim etapie estymowana jest wartość wielkości mierzonej. Pierwszy etap jest realizowany układowo. Drugi etap jest realizowany programowo. Wynik pomiaru jest otrzymywany metodami cyfrowego przetwarzania danych.

Niepewność zaewidencjonowanych danych przekłada się na niepewność wyniku estymacji, który jest wynikiem pomiaru. Zastosowanie w systemie akwizycji DAQ konwersji a-c z sygnałem diterowym, modyfikuje błąd kwantowania tak, iż niepewność ta ulega redukcji. Modyfikacja ta polega na randomizacji błędu kwantowania, który może być dzięki temu wyeliminowany w wyniku uśredniania lub filtracji. Randomizacja może być częściowa lub całkowita. Stopień randomizacji i eliminacji błędu kwantowania zależy od możliwości dobrania sygnału ditherowego gaussowskiego o odpowiednim odchyleniu standardowym (w relacji do rozdzielczości zastosowanego przetwornika A/C) lub sygnału ditherowego o prostokątnej funkcji gęstości prawdopodobieństwa i o odpowiednim zakresie zmian wartości (w relacji do rozdzielczości zastosowanego przetwornika A/C).

Dla każdego z trzech przypadków: DAQ z konwersją a-c, DAQ z konwersją a-c i sygnałem ditherowym, DAQ z konwersją a-c i sygnałem ditherowym optymalnym, podano zależności określające złożoną niepewność standardową wynikającą z błędów modułu DAQ. Uzasadniono (porównując te zależności), iż zastosowanie konwersji a-c z sygnałem ditherowym dobranym optymalnie lub nie, w każdym przypadku powoduje zmniejszenie niepewności wyniku pomiaru.