METROLOGIA I SYSTEMY POMIAROWE
TOM V, ZESZYT 4 (1998)

STEFAN KUBISA

Institute of Control Engineering, Technical University of Szczecin

ERROR DISTRIBUTION OF A SET OF MEASURING INSTRUMENT AND AN INFLUENCE OF "STEP BY STEP" CALIBRATION PROCEDURE ON THE DISTRIBUTION

The hypotheses of an error probability distribution of a measuring instrument inspire a lot of discussions. A basis of the considerations, presented in this paper, is a discrete, step by step calibration procedure of a simple measuring instrument (a simple-value measure, simple measuring element). The paper shows that such procedure leads to the rectangular error distribution of the simple instrument, if some metrological as well as some economical demands are met. The distribution may be normal in a case of a complex instrument, when the values of the readings or settings of the instrument are a result of addition of many values represented by some elements calibrated separately. A selection procedure of the elements may lead to a bi-rectangular error distribution of these elements. Every calibration and selection procedure is disturbed by some random factors in reality, so the distribution of the instrument error has not only the rectangular or/and bi-rectangular components, but a normal component as well.

1. INTRODUCTION

Probabilistic approach to the description of the real world events is dominating in science presently. It is observed in the international recommendation guide describing the way of the measurement inaccuracy expression (*Guide to the Expression of Uncertainty in Measurement* [1]). The *Guide* describes in the probabilistic way not only a component of the measurement uncertainty caused by some random effects, but an uncertainty component caused by the inaccuracy of the measuring instrument. The latter statement is controversial, because the instrument error is classified traditionally to a group of systematic errors.

The *Guide's* approach to the problem is not new. A similar approach was presented by S. Trzetrzewiński in lecture [2] given at a metrologist meeting in Wrocław over 40 years ago. The author of this paper owns the text of it, unfortunately without its full bibliographic data.

Interpretational difficulties connected with probabilistic approach of the instrument error is discussed by J. M. Jaworski [3]. He introduces the idea of randomisation and centring of this error by means of a mental experiment. This experiment might be imagined as a series of observations made by a different, randomly chosen sample of the measuring instrument. J. M. Jaworski emphasises, that this method is not used in practice because it contradicts the rule of stability of measuring experiment conditions, and is expensive.

The author has the following solution of this difficulty: having a sample of the measuring instrument is a random event which randomises the instrument error. (The point of this random event is, that the instrument sample we have is one of those produced and sold). It is assumed, that the instrument sample is an element of some family of the instruments satisfying defined metrological demands. A reading error of this instrument for each specified measured value is a random variable of the instruments family. The variable is characterised by some probability density distribution.

A calibration procedure of the instrument influences the distribution. "Step by step" calibration is one of the possible procedure. The procedure is relatively simple. The paper shows, how the parameters of the procedure, possibly together with a selection procedure, influence the considered probability distribution.

2. MODEL OF "STEP BY STEP" CALIBRATION PROCEDURE

The aim of a calibration procedure is to cause, that the instrument error of the calibrated instrument Δ_M be contained in the interval of the acceptable limit error L after the calibration procedure is finished:

$$-L<\Delta_M<L \tag{1}$$

The "step by step" calibration procedure is a discrete process, realised in M steps. Generally, the m-th step of the procedure ($m=0, 1, 2, ..., M$) includes:

— settlement of the present value Δ_m of instrument error,

— checking its relation with the acceptable error L,

— deciding whether to end the procedure or to introduce the reading correction of the calibrated instrument in the proper direction.

In the successive steps the error has values $\Delta_0, \Delta_1, \Delta_2, ..., \Delta_m, \Delta_{m+1}, ..., \Delta_M$ differing in correction step value Δ_{st} among each other:

$$|\Delta_{m+1}-\Delta_m|=\Delta_{st}>0 \tag{2}$$

This is illustrated in Fig. 1. The figure explains, that the correction step should satisfy the condition:

$$0<\Delta_{st}\leqslant 2\cdot L \tag{3}$$

in order to finish each calibration successfully, i.e. to reach the calibration aim (1) independently from the initial error value Δ_0 — instrument error value of the uncalibrated instrument.

The calibration procedure can be made shorter, if the number M of the correction steps is calculated earlier on the basis of the initial error value Δ_0 and the acceptable

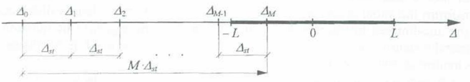


Fig. 1. Illustration of a "step by step" calibration procedure of a measurement instrument

error value L and if the M steps is realised simultaneously. However this shortening of the procedure does not influence the final distribution of the instrument error of the set of instruments.

3. TRANSFORMATION OF ERROR PROBABILITY DISTRIBUTION IN "STEP BY STEP" CALIBRATION PROCEDURE

The initial value of the instrument error Δ_0 is a random variable of the family of the instruments before calibration and it has a probability density distribution $g_0(\Delta_0)$, which will be called an initial function. Let us assume that this initial function is known. The calibration procedure transforms the initial value Δ_0 of the instrument

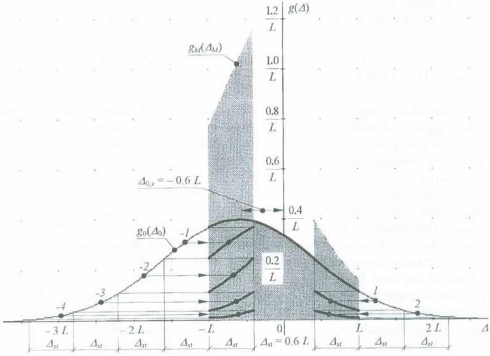


Fig. 2. Transformation of the initial function $g_0(\Delta_0)$ into the final function $g_M(\Delta_M)$

error into a final value Δ_M, which satisfies the condition (1). The procedure also transforms the initial function $g_0(\Delta_0)$ (the function of probability density distribution of the uncalibrated instrument error) into a final function $g_M(\Delta_M)$ (the function of probability density distribution of the calibrated instrument error). Fig. 2 explains the mechanism of this transformation:

1) A segment of the initial function $g_0(\Delta_0)$ graph contained in the interval $(-L, L)$ and marked with thick line is still contained in the interval.

2) The segments of the initial function $g_0(\Delta_0)$ graph contained in the following intervals of the correction step length Δ_{st}, marked with symbols -1, -2, ..., are shifted right to the interval $(-L, -L+\Delta_{st})$.

3) The segments of the initial function $g_0(\Delta_0)$ graph contained in the following intervals of the correction step length Δ_{st}, marked with symbols 1, 2, ..., are shifted left to the interval $(L-\Delta_{st}, L)$.

4) The sum of ordinates of all segments mentioned in 1), 2) and 3) create the final function $g_M(\Delta_M)$ values in the interval $(-L, L)$. The final function has value zero beyond this interval.

The form of the final function $g_M(\Delta_M)$ (the function of probability density distribution of the calibrated instrument error) depends on:

— the form of the initial function $g_0(\Delta_0)$ (the function of probability density distribution of the uncalibrated instrument error),

— the acceptable error value L,

— the calibration procedure parameter — the correction step Δ_{st}.

4. ANALYSIS OF THE ERROR DISTRIBUTION OF THE CALIBRATED INSTRUMENT (THE FINAL FUNCTION)

The probability density distribution of the calibrated instrument error is the final function $g_M(\Delta_M)$ of many parameters. Therefore, the variability intervals of these, parameters should be limited to the real (from the practical point of view) values in order not to consume much time for analysing the function and to receive clear and useful results.

It can be assumed, the initial error Δ_0 has a normal distribution with an expected value $\Delta_{0,e}$ and with a standard deviation σ_0 — the initial function $g_0(\Delta_0)$ is a Gaussian function. This assumption is not controversial, because the errors of the instruments before calibration depend on many random factors of their manufacturing process.

The calibration procedure analysis is illustrated in Fig. 2, which also shows, that the form of the final function $g_M(\Delta_M)$ depends on the form of the initial function $g_0(\Delta_0)$ not so much, provided the initial function is smooth and sufficiently "broad". This "breadth" means sufficiently high value of the initial standard deviation σ_0 in relation to the acceptable error L. For example, the initial standard deviation σ_0 a few times smaller than the acceptable error L means high repeatability of the instrument manufacturing process and generally eliminates the necessity of the calibration because the repeatable manufacturing process enables one to receive sufficiently low (according to modulus) expected value $\Delta_{0,e}$ of the initial error Δ_0. For the purpose of numerical analysis the following values of the initial standard deviation σ_0 are used: $0.7 \cdot L$, L, $7 \cdot L$.

The expected value $\Delta_{0,e}$ and the standard deviation σ_0 of the initial error Δ_0 decide together, whether the procedure of the instrument error correction would run in the same direction for each instrument sample (one way procedure) or, randomly, in one or opposite direction (two ways procedure). The one way procedure is usually more comfortable from the technical point of view, because it needs simpler means of the error correction in many cases, for example a laser cutting of electronic system pathways or removing an excess material in case of weights or gauge blocks.

The fulfilment of the inequality:

$$|\Delta_{0,e}| > (2 \ldots 3) \cdot \sigma_0 \tag{4}$$

is the practical condition of the one way procedure, i.e. the initial error Δ_0 of the normal distribution should be almost ever of the same sign. This inequality gives the probability of one way procedure over (95 ... 99.7)%.

It is assumed for numerical analysis, that the expected value $\Delta_{0,e}$ of the initial error Δ_0 is either equal to zero or it is positive (the error is corrected left in calibration procedure) and has a value being equal to L. It is about $1.43 \cdot \sigma_0$ for the lowest value of $\sigma_0 = 0.7 \cdot L$, which is used in the analysis of an influence of the expected value $\Delta_{0,e}$. Increase of the expected value $\Delta_{0,e}$ does not change the analysis result significantly and the negative expected values $\Delta_{0,e}$ give the "mirror reflex" values of these values from the positive side.

A domain of the correction step Δ_{st} value is defined by the condition (3). The following values of the step Δ_{st} for the analysis: $0.5 \cdot L$, L, $1.5 \cdot L$, $2 \cdot L$ has been assumed.

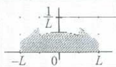
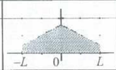
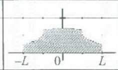
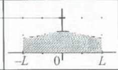
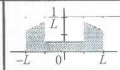
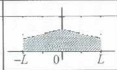
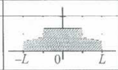
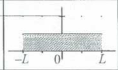
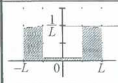
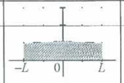
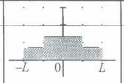
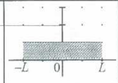
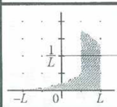
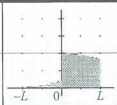
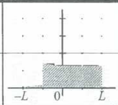
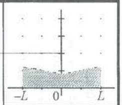
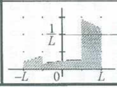
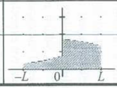
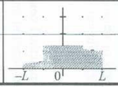
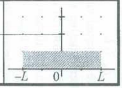
The rows 1, 2 and 3 in the Table 1 show twelve final function graphs for the zero expected value $\Delta_{0,e}$ of the initial error Δ_0, for three values of the initial standard deviation σ_0: $0.7 \cdot L$, $1.4 \cdot L$, $7 \cdot L$ and for four values of the correction step Δ_{st} given above. This study of the form of the final function $g_M(\Delta_M)$ allows one to create the following conclusions connected with the case where $\Delta_{0,e} = 0$:

I. The low values of the correction step ($\Delta_{st} < L$) give (at least for $\Delta_{0,e} = 0$) the bimodal final functions $g_M(\Delta_M)$ (compare with Fig. 2); probability density is high for the final error Δ_M value near to $-L$ and L. It is not advantageous.

II. If the correction step equals the acceptable error ($\Delta_{st} = L$) the final function $g_M(\Delta_M)$ (at least for $\Delta_{0,e} = 0$) tends towards a rectangular distribution, if the initial standard deviation σ_0 increases (if $\sigma_0 \to \infty$).

III. The especially important case is, when the correction step equals the double acceptable error ($\Delta_{st} = 2 \cdot L$). The final function $g_M(\Delta_M)$ is then near (more detailed analysis — see further) to the function of the rectangular distribution for sufficient high values of the initial standard deviation σ_0.

T a b l e 1. The examples of the graphs of instrument error probability distribution

		$\Delta_{st} = 0.5 \cdot L$	$\Delta_{st} = L$	$\Delta_{st} = 1.5 \cdot L$	$\Delta_{st} = 2 \cdot L$
$\Delta_{0,e} = 0$	$\sigma_0 = 0.7 \cdot L$				
	$\sigma_0 = 1.4 \cdot L$				
	$\sigma_0 = 7 \cdot L$				
$\Delta_{0,e} = L$	$\sigma_0 = 0.7 \cdot L$				
	$\sigma_0 = 1.4 \cdot L$				

IV. Specific features of the form of final function graph are emphasised by relatively low values of the initial standard deviation σ_0.

The analysis of the influence of the expected value $\Delta_{0,e}$ on the final function $g_M(\Delta_M)$ is made for two initial standard deviation values $\sigma_0=0.7\cdot L$ and $\sigma_0=1.4\cdot L$. The rows 4 and 5 of the Table 1 show the results of this analysis in form of eight final function $g_M(\Delta_M)$ graphs — for one expected value $\Delta_{0,e}$ of the initial error Δ_0, i.e. for $\Delta_{0,e}=L$ and for four values of the correction step $\Delta_{st}=0.5\cdot L$, L, $1.5\cdot L$, $2\cdot L$. The results confirms the general charàcter of the conclusions III. and IV. and they allow one to formulate the following conclusion:

V. Non-zero expected value $\Delta_{0,e}$ of the initial error Δ_0 causes asymmetry of the final function $g_M(\Delta_M)$, especially by low correction step $\Delta_{st}<2$. Here appears the higher probability of the error of the same sign as the sign of the expected value $\Delta_{0,e}$. The expected value $\Delta_{M,e}$ of the final error Δ_M is different from zero. This is an disadvantageous event, which disappears when the correction step values Δ_{st} tends to double value of the acceptable error L.

The conclusion V. demands more detailed analysis of the final function $g_M(\Delta_M)$ according to the expected value $\Delta_{M,e}$ of the final error Δ_M.

5. EXPECTED VALUE OF THE FINAL ERROR AND ITS CONVERGENCE TO THE RECTANGULAR DISTRIBUTION

The above analyses show, that the ratio $\Delta_{M,e}/L$ of the expected value $\Delta_{M,e}$ of the final error Δ_M (the calibrated instrument error) to the acceptable value L is a function of three variables, i.e. the ratios $\Delta_{0,e}/L$, Δ_{st}/L, σ_0/L. The graphs of this function are shown in Fig. 3. The figure shows two families of curves: for $\sigma_0/L=0.7$ and $\sigma_0/L=2$ and for $\Delta_{st}/L=0.5, 1, 1.5, 2$. The ratio $\Delta_{0,e}/L$ of the expected value $\Delta_{0,e}$ of the initial error Δ_0 to the acceptable error L is chosen as a continuos variable. The examined function is odd in relation to this variable $\Delta_{0,e}/L$ — a zero-point of the coordinate system is the middle of symmetry of the graphs and therefore the graphs are made only for the positive values of this variable. The analysis of this graphs leads to the following conclusions:

VI. The expected value $\Delta_{M,e}$ of the final error Δ_M is strictly zero, if the expected value $\Delta_{0,e}$ of the initial error Δ_0 is zero. At the same time this value $\Delta_{M,e}$ tends towards zero, if correction step Δ_{st} tends towards the double value of the acceptable error L, especially for sufficient high value of the initial standard deviation σ_0.

VII. If the correction step Δ_{st} value is near to the double value of acceptable error L, then an oscillatory character of the expected value $\Delta_{M,e}$ as a function of the ratio $\Delta_{0,e}/L$ is noticed, especially for the low values of the initial standard deviation σ_0. These oscillations are a result of a wavy form of the final function $g_M(\Delta_M)$ of the final error Δ_M and a result of shifting of this "wave" by the changes of the expected values $\Delta_{0,e}$ — what can be observed on the graphs in the last column of table 1 (for $\Delta_{st}=2\cdot L$, $\sigma_0=0.7\cdot L$).

VIII. The oscillation amplitude of the expected value $\Delta_{M,e}$ and the "waving" amplitude of the final function $g_M(\Delta_M)$ (discussed in conclusion VII.) decreases

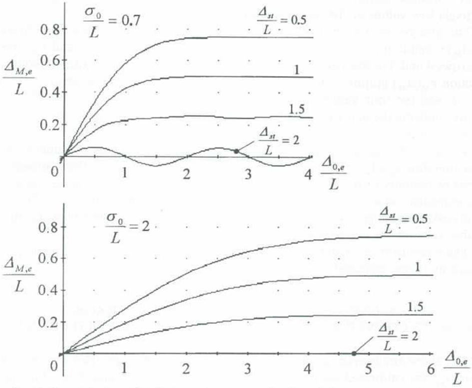


Fig. 3. Expected value of the instrument error as a function of the parameters: $\Delta_{0,e}$, Δ_{st}, σ_0

rapidly if the initial standard deviation σ_0 increases and then the final function $g_M(\Delta_M)$ tends towards a function of the rectangular distribution.

So the probability density distribution $g_M(\Delta_M)$ of the final error Δ_M (the calibrated instrument error) is convergent to the rectangular distribution, if the correction step Δ_{st} tends towards the double value of acceptable error L and if the standard deviation σ_0 of the initial error Δ_0 is sufficiently high. The latter condition should be met in real situations, what was discussed already in Chapter 4. The correction step Δ_{st} value is a result of a technologist decision, who determines the details of the calibration procedure. If the technologist takes under consideration the economical aspects of the procedure, then he should minimalise the calibration time — i.e. to minimalise the necessary number of the calibration steps. It should lead him to decision: $\Delta_{st} = 2 \cdot L$!

6. SPECIAL CASES. MEASUREMENT INSTRUMENT SELECTION AND THE SELECTION RESULT — A BI-RECTANGULAR DISTRIBUTION

It was proved above, that the error probability density distribution of the calibrated measuring instrument is close to the rectangular distribution, if the rational correction step $\Delta_{st}=2 \cdot L$, motivated by metrological and economical aspects is chosen. The assuming of this instrument error distribution [4—11] seems to be proper in many cases, especially, if the succeeding instrument readings or settings undergo the calibration procedure.

Another situation arises, when the values of the instrument readings or settings are a result of addition of many values represented by some elements calibrated separately. Such situation can refer especially the measures in form of a set, for example a set of the weights or the gauge blocks. It can refer also to the multi-value measures of the electric quantities as the resistors and capacitors set step by step i.e. the decade resistors and capacitors, if each element of these multi-value measures has been calibrated individually, not together in the set. The error distributions of the separated set elements are rectangular (or bi-rectangular — see further). The distribution of the instrument total error (referring to the value set on the instrument) — the combined distribution — is a convolution of the distributions of the elements used to create the set value. This combined distribution tends towards a normal distribution, if there is a big number of elements creating the set value — i.e., if the conditions of the Central Limit Theorem are met.

It can be possible to reach a proper accuracy of a single value measure or a proper accuracy of the elements of a multi-value measure by making selection. The elements, which errors are contained in the limits of the acceptable error l lower than the acceptable error L ($l<L$), can be chosen from the family of the elements of the acceptable error L (the source family). This selection divides the source family of the acceptable error L onto two disjoint families:

— a family of the elements of a higher accuracy class (with the acceptable error l) — the family cutting out from the source family,

— a remainder of the source family — a remaining family of the elements of lower accuracy class (with the acceptable error L).

If the source family is characterised by rectangular distribution (with half breadth L), then the family of the elements of the higher accuracy class has also the rectangular distribution, but with half breadth l. At the same time the remaining

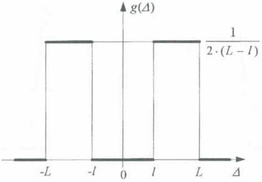


Fig. 4. Bi-rectangular distribution

family of the elements of lower accuracy class is characterised by bi-rectangular distribution [4, 5], shown in Fig. 4. The acceptable errors for the following accuracy classes are usually in relation near to 1 : 2, i.e. $l/L \approx 1/2$. A bi-rectangular distribution with $l/L = 1/2$ is called a special bi-rectangular distribution [4].

7. THE RANDOM EVENTS ATTENDANT UPON THE CALIBRATION AND THEIR INFLUENCE ON THE INSTRUMENT ERROR DISTRIBUTION

The shown above idealised "step by step" procedure of calibration leads to rectangular error distribution of a family of simple measuring instrument by the rational choose of the correction step. The selection procedure leads to the bi-rectangular distribution. Every selection and calibration procedure is disturbed by factors having a random character in reality. Those factors increase the instrument error of a component which can be attributed to a normal distribution. The combined distribution of the total instrument error is, by this assuming, a convolution of

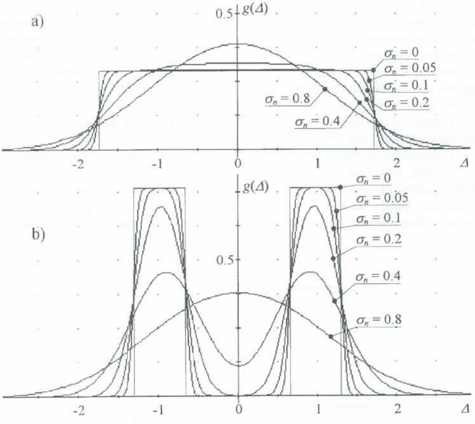


Fig. 5. Influence of the random events on the instrument error distribution: a) — on the rectangular distribution, b) — on the bi-rectangular distribution

a distribution, resulting from the calibration or from the calibration and selection procedure — the rectangular or bi-rectangular distribution — with the normal one.

A study of the graphs of this convolution is shown in Fig. 5. Fig. 5a shows the graphs of the normal-rectangular convolution, Fig. 5b — the graphs of the normal-bi-rectangular convolution. The graphs are standardised (the convolution standard deviation σ has value $\sigma=1$). A relative value of a standard deviation σ_n of the normal distributed error component (a ratio σ_n/σ of the mentioned standard deviations) is a parameter of the graphs. The graphs are smooth already by a very low value of the ratio σ_n/σ, i.e. 0.05. If the ratio is near 1 — i.e. 0.8 — the convolution graph has a form similar to the normal distribution graph.

The separate, not considered in this paper, is the question, how high can be really the value of the ratio σ_n/σ.

8. CONCLUSION

The paper shows, that the idealised "step by step" calibration procedure of measuring instrument, where the rational metrological and economical aspects are taken under consideration, leads to rectangular distribution of probability density of instrument error. The selection procedure can cause a bi-rectangular distribution (Fig. 4). These distributions can be changed by some random events, present by the calibration or selection (Fig. 5).

Notation

Δ_M — instrument error after the instrument calibration
L — acceptable limit instrument error after the calibration
M — total number of the calibration steps
m — m-th step of the calibration procedure ($m=0, 1, 2, ..., M$)
Δ_m — instrument error in m-th step of the calibration
Δ_0 — instrument error before calibration – instrument error in 0-th step of the calibration
Δ_{st} — length of the calibration step
$g_0(\Delta_0)$ — probability density function of the error Δ_0
$g_M(\Delta_M)$ — probability density function of the error Δ_M
σ_0 — standard deviation of the error Δ_0
$\Delta_{0,e}$ — expected value of the error Δ_0
$\Delta_{M,e}$ — expected value of the error Δ_M
l — acceptable limit error of a higher class instrument
σ — standard deviation of a convolution of the probability density functions
σ_n — standard deviation of a normal distributed component of the instrument error.

REFERENCES

1. *Guide to the Expression of Uncertainty in Measurement.* ISO/IEC/OIML/BIPM, first edition (1992).
2. Trzetrzewiński S.: *Dokładność pomiarów elektrycznych (Accuracy of the Electrical Measurements).* A typescript of a lecture given at a metrologist meeting in Wrocław, Poland, in the 50-ties, 24 pages.
3. Jaworski J.M.: *Niedokładność, błąd, niepewność (Inaccuracy, Error, Uncertainty).* Proc. of XXIX Międzyuczelniana Konferencja Metrologów, 10–12 September 1997, Nałęczów–Lublin (1997), pp. 197–216.
4. Kubisa S.: *Geometric Sum of the Expanded Uncertainties as an Estimate of the Combined Expanded Uncertainty in Measurement.* Metrology and Measuring Systems, III, 3–4, Warsaw (1997), pp. 157–167.
5. Kubisa S.: *Generalised Geometric Sum of the Expanded Uncertainties as an Estimate of Combined Expanded Uncertainty.* Proc. of the 4th International Symposium on Methods and Models in Automation and Robotics, 26–29 Aug. 1997, Międzyzdroje, Poland, V. 2, pp. 593–598.
6. Kubisa S., Turzeniecka D.: *Evaluation of some Approximated Methods of Measurement Uncertainty Estimation.* Proc. of the Third International Symposium on Methods and Models in Automation and Robotics, 10–13 September 1996, Międzyzdroje, Poland, pp. 537–542.
7. Turzeniecka D.: *Relations Between Selected Uncertainties and Their Influence on the Accuracy of Various Evaluation.* Proc. of the 4th International Symposium on Methods and Models in Automation and Robotics, 26–29 Aug. 1997, Międzyzdroje, Poland, V. 2, pp. 573–578.
8. Turzeniecka D., Kubisa S.: *Error and Uncertainty in Measurement — Teaching Problems.* Proc. of 3rd East-West Congress on Engineering Education, 15–20 September 1996, Gdynia, Poland, pp. 118–122.
9. Turzeniecka D., Kubisa S.: *The Measures of Imperfection of Chosen Approximated Methods of Combined Expanded Uncertainty Estimation in Measurement.* Metrology and Measuring Systems, III, 3–4, Warsaw (1997), pp. 143–155.
10. Turzeniecka D., Waśkiewicz Z.: *Analiza sytuacji niejednoznacznych w ocenie niepewności wyniku pomiarów.* Metrology and Measuring Systems, 1, Warsaw (1996).
11. Turzeniecka D., Waśkiewicz Z.: *Wybrane problemy wspomaganej komputerowo statystycznej oceny niepewności pomiaru.* Proc. of VII Krajowa Konferencja Metrologii, t. I. Politechnika Warszawska, Warsaw (1995), pp. 135–142.

ROZKŁAD PRAWDOPODOBIEŃSTWA BŁĘDU APARATUROWEGO W POPULACJI PRZYRZĄDÓW I WPŁYW KROKOWEJ PROCEDURY KALIBRACJI NA TEN ROZKŁAD

Streszczenie

Hipotezy dotyczące rozkładu prawdopodobieństwa błędu aparaturowego wywołują wiele dyskusji. Podstawą do rozważań przedstawionych w tym artykule jest dyskretna, krokowa procedura kalibracji prostego przyrządu pomiarowego (jednowartościowej miary, pojedynczego elementu pomiarowego). W artykule wykazano, że taka procedura prowadzi do jednostajnego rozkładu prawdopodobieństwa błędu prostego przyrządu pomiarowego, jeżeli spełnione są pewne wymagania metrologiczne i ekonomiczne. Rozpatrywany rozkład może też być normalny w przypadku złożonego przyrządu pomiarowego — gdy wartości jego odczytów lub nastaw są wynikiem dodawania wielu wartości reprezentowanych przez pewne elementy kalibrowane oddzielnie. Procedura selekcji tych elementów może też prowadzić do bi-jednostajnego rozkładu ich błędu. Każda procedura kalibracji i selekcji w realnych warunkach zakłócona jest czynnikami przypadkowymi. Powodują one, że rozkład błędu aparaturowego ma składową normalną, a nie tylko składowe jednostajne i bi-jednostajne.